Dynamic Hedging of Credit Derivative: a First Approach*

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Abstract

In this note, we show on a stylised example how one can hedge Basket Credit Derivatives using a related family of liquid hedging products. Using simple Non-Arbitrage arguments and results from stochastic calculus, we prove that one can build a self-financing portfolio written on Credit Default Swaps which replicates the payoff of a general Credit Derivative.

1 Introduction

To price more and more complex Basket Credit Derivatives academic and practitioners have developped a wide range of model which can roughly be split into two categories: the so-called structural (corresponding to the celebrated firm value approach to credit, see [B-R] for an overview) or reduced-form model (see [A-W] or [SCH] for two different ways to specify such a model; see also [GRO] and reference therein).

We work in the reduced-form model framework where the default times of the reference entities are defined using exogenous random variables and do not have links with economical variables. We will consider that our model is specify as soon as the joint law of the default times is chosen. We also assume that the interest rates are deterministic which is equivalent to assume that the reference filtration is trivial.

Despite its obvious practical interest, the Hedging of Basket Credit Derivatives has not been intensively studied yet. There is several reasons to explain such a situation: one could mention for instance the lack of liquid products due to the youth of the market but also the prohibitive cost of a perfect hedging.

We nevertheless adress this issue in this paper although from a theoretical point of view. More precisely, we shall prove that under the assumption of the existence of liquid Credit Default Swap on each reference entity of a two-firm basket one can use them as basic underlying to write self-financing portfolio which replicates any payoff. Our approach provides a model-coherent way to hedge a Basket Credit Derivatives: indeed it take into account the information about all obligors and the way it is revealed to us as time goes by.

After describing our modelling hypothesis, we prove a martingale representation theorem that will be the key of the subsequent development. This result is then use to prove the main result of this paper which can be expressed in the following way: under mild regularity assumptions the reduced-form model with trivial reference filtration is *complete* with respect to Credit Default Swap. We end this note with some numerical applications.

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1.1 Modelling Default Times

There are several way to prescribe the dependency between reference entities. One may cite Copula Models [SCH] or Credit Contagion Model [A-W]. We consider here a model on two reference entities F_1 and F_2 which is specified by the choice of the joint risk-neutral law of the default times (τ_1, τ_2) of the two considered firms. In other words, we suppose given a survival function G defined by

$$G(t, s) = \mathbb{P}[\tau_1 > t\,;\, \tau_2 > s].$$

We made the technical assumption that this survival function is smooth and we assume throughout this note that the instantaneous interest rates r are deterministics. Up to an immediate change of numéraire, we may even assume that there are equal to 0. Information avalaible at date t is modelized by the σ-field $\mathcal{G}_t$ generated by the random variables $\min(\tau_1, t)$ and $\min(\tau_2, t)$. We will implicitely assume that this filtration satisfies the usual hypothesis[1].

1.2 Contracts Features and Risk-Neutral Valuation

We will consider three contracts on the Credit Basket $\{F_1, F_2\}$: two Credit Default Swaps and an exotic credit derivatives hereafter called CD. For sake of simplicity only[2], we will assume that: recovery rates are equal to zero (digital CDS), buyer's margin is paid upfront (which means at the beginning of the contract) and in case of default the covered notional is paid at maturity. The payoff of the considered CD is given by

$$C_1 I_{\{\tau_{(1)} \le T\}} + C_2 I_{\{\tau_{(2)} \le T\}}$$

where $(\tau_{(1)}, \tau_{(2)})$ is the order statistic vector of τ such that $\tau_{(1)} < \tau_{(2)}$.

Using standard Arbitrage-Free arguments (see for instance [B-R]), one may prove that the value (from the point of view of the protection's buyer) at the date t of the different contracts are given by

$$CDS^i(t) = \mathbb{E}[I_{\{0<\tau_i \le T\}}|\mathcal{G}_t], \qquad (i = 1, 2), \tag{1}$$

$$CD(t) = \mathbb{E}\big[C_1 I_{\{0<\tau_{(1)} \le T\}} + C_2 I_{\{0<\tau_{(2)} \le T\}}|\mathcal{G}_t\big]. \tag{2}$$

2 Mathematical Preliminaries

In this section, we treat the mathematical part of the paper. One will find all the necessary computation and a primer on the stochastic calculus results that are used in the sequel.

2.1 Conditional Expectation Computations

In this paragraph, we give hints about how one can compute conditional expectations of the form $\mathbb{E}[f(\tau_1, \tau_2)|\mathcal{G}_t]$. The first step is to write that:

$$\begin{aligned}\mathbb{E}[f(\tau)|\mathcal{G}_t] = \mathbf{1}_{\{\tau_1 \le t\,;\,\tau_2 \le t\}} f(\tau_1, \tau_2) + \mathbf{1}_{\{\tau_1 \le t\,;\,\tau_2 > t\}} \mathbb{E}[f(\tau)|\tau_1\,,\, \tau_2 > t] \\ + \mathbf{1}_{\{\tau_1 > t\,;\,\tau_2 \le t\}} \mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2] + \mathbf{1}_{\{\tau_1 > t\,;\,\tau_2 > t\}} \mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2 > t],\end{aligned}$$

or, in other words,

$$\begin{aligned}\mathbb{E}[f(\tau)|\mathcal{G}_t] = J_t(0,0)(\tau_1, \tau_2) f(\tau_1, \tau_2) + J_t(0,1)(\tau_1, \tau_2) \mathbb{E}[f(\tau)|\tau_1\,,\, \tau_2 > t] \\ + J_t(1,0)(\tau_1, \tau_2) \mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2] + J_t(1,1)(\tau_1, \tau_2) \mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2 > t],\end{aligned}$$

[1] which are completeness and right-continuity. These hypothesis are satisfied in our case up to the addition to each $\mathcal{G}_t$ of the $\mathbb{P}$-negligible sets.

[2] as these hypothesis can easily be relaxed to reach more general cases.

where if $I_t(0) = [0,t]$, $I_t(1) =]t,+\infty[$ and $\epsilon = (\epsilon_1,\epsilon_2) \in \{0,1\}^2$, $J_t(\epsilon)$ denote the indicator function of the set $I_t(\epsilon_1) \times I_t(\epsilon_2)$. Then, using the following computational trick

$$\mathbb{E}[f(\tau)|\tau_1 = s\,,\, \tau_2 > t] = \frac{\partial_s \mathbb{E}[f(\tau_1,\tau_2)\mathbf{1}_{\{\tau_1>s\,;\,\tau_2>t\}}]}{\partial_s \mathbb{P}[\tau_1 > s\,;\, \tau_2 > t]}$$

one may easily prove that

$$\begin{aligned}
\mathbb{E}[f(\tau)|\tau_1 = t, \tau_2 = s] &= E\big((0,0),t,s,f\big) = f(t,s),\\
\mathbb{E}[f(\tau)|\tau_1 = s\,,\, \tau_2 > t] &= E\big((0,1),t,s,f\big) = \int_t^\infty f(s,v)\frac{(\partial_1 G)(s,\mathrm{d}v)}{(\partial_1 G)(s,t)},\\
\mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2 = s] &= E\big((1,0),t,s,f\big) = \int_t^\infty f(u,s)\frac{(\partial_2 G)(\mathrm{d}u,s)}{(\partial_2 G)(t,s)}\,,\\
\mathbb{E}[f(\tau)|\tau_1 > t\,,\, \tau_2 > s] &= E\big((1,1),t,s,f\big) = \int_t^\infty \int_s^\infty f(u,v)\frac{G(\mathrm{d}u,\mathrm{d}v)}{G(t,s)},
\end{aligned}$$

and finally, putting all these building blocks together, one find that

$$\mathbb{E}[f(\tau_1,\tau_2)|\mathcal{G}_t] = \sum_{\epsilon\in\{0,1\}^2} J_t(\epsilon)(\tau)E(\epsilon,\tau\wedge t,f).$$

Using this result, we made other computations that are useful for numerical applications: here are some formulas given without any proof:

$$\begin{aligned}
CDS^1(t) &= I_{\{\tau_1>t\}}\Big(1 - I_{\{\tau_2\le t\}}\frac{(\partial_2 G)(T,\tau_2)}{(\partial_2 G)(t,\tau_2)} - I_{\{\tau_2>t\}}\frac{G(T,t)}{G(t,t)}\Big) + I_{\{\tau_1\le t\}},\\
CDS^2(t) &= I_{\{\tau_2>t\}}\Big(1 - I_{\{\tau_1\le t\}}\frac{(\partial_1 G)(\tau_1,T)}{(\partial_1 G)(\tau_1,t)} - I_{\{\tau_1>t\}}\frac{G(t,T)}{G(t,t)}\Big) + I_{\{\tau_2\le t\}},
\end{aligned}$$

$$\begin{aligned}
CD(t) = C_1 I_{\{\tau_{(1)}>t\}}\Big[1-\frac{G(T,T)}{G(t,t)}\Big] &+ C_2 I_{\{\tau_{(2)}>t\}} + C_1 I_{\{\tau_{(1)}\le t\}}\\
&\times\Big[I_{\{\tau_1>t\,;\,\tau_2\le t\}}\Big(1-\frac{(\partial_2 G)(T,\tau_2)}{(\partial_2 G)(t,\tau_2)}\Big)\\
&+ I_{\{\tau_1\le t\,;\,\tau_2>t\}}\Big(1-\frac{(\partial_1 G)(\tau_1,T)}{(\partial_1 G)(\tau_1,t)}\Big)\\
&+ I_{\{\tau_1>t\,;\,\tau_2>t\}}\Big(1-\frac{G(t,T)+G(T,t)-G(T,T)}{G(t,t)}\Big)\Big]\\
&+ C_2 I_{\{\tau_{(2)}\le t\}}.
\end{aligned}$$

2.2 Some Classical Stochastic Calculus Results

We make use in the text of some classical probabilistic results. We state them properly in this paragraph. The full details may be found in the excellent book from Rogers & Williams [R-W]. Let $(\Omega,\mathcal{F},\mathbb{P},\mathbb{F} = (\mathcal{F}_t)_{t\ge 0})$ be a complete filtered probability space satisfying the usual conditions which means that the filtration $\mathbb{F}$ is complete and right-continuous. Recall that if $(L_t)_{t\ge 0}$ is a càdlàg[3] process, we denote by $\Delta L_t = L_t - L_{t-}$, the jump process of L. In this setting, the following results are true:

PROPOSITION 1. *Let $\tau_1,\dots,\tau_N$ be N stopping times*[4] *of the filtration $\mathbb{F}$ and $X_1,\dots,X_N$ be bounded (or more generally integrable) random variable such that, for all n, X_n is $\mathcal{F}_{\tau_n}$*[5]*-measurable. There is a unique $\mathbb{F}$-martingale M whose jump process is equal to*

$$t\mapsto \Delta\Big(\sum_{n=1}^N X_n \mathbf{1}_{]\!]0,\tau_n]\!]}(t)\Big).$$

[3] which means right-continuous with left-limit.
[4] A random variable τ is a $\mathbb{F}$-stopping time as soon as $\{\tau\le t\}$ is $\mathcal{F}_t$-measurable for all t.
[5] Recall that if τ is a stopping time, $\mathcal{F}_\tau$ is the σ-field $\{A\,;\, A\cap\{\tau\le t\}\in\mathcal{F}_t, \forall t\}$.

Furthermore, if M^i denote the unique martingale such that $\Delta M^i = \Delta I_{\rrbracket 0,\tau_i \rrbracket}$, there exists previsible[6] *processes H^i such that $M = \sum_i \int H_i \, \mathrm{d}M^i$ and these processes are characterized by the property $H_i(\tau_i) = X_i$.*

2.3 Martingale Representation

We prove here the main mathematical result. It appear that despite (or maybe because...) its simplicity, we do not find any explicit statement of this result in the existing litterature.

For $i = 1, 2$, let M^i be the unique $\mathcal{G}$-martingale such that the jump processes $\Delta M^i_t := M^i_t - M^i_{t-}$ are equal to $\Delta I_{\{\tau_i \leq t\}}$. We will make use of the following representation result:

PROPOSITION 2. *Let f be a bounded measurable function and M^f the martingale $t \mapsto \mathbb{E}[f(\tau)|\mathcal{G}_t]$. Then there exists a two-dimensional $\mathcal{G}$-previsible process $H = (H^f_1, H^f_2)$ such that*

$$M^f_t = \mathbb{E}[f(\tau)] + \int_{]0,t]} H^f(s) \cdot \mathrm{d}M_s \,. \tag{3}$$

Furthermore, H^f_i is the unique $\mathcal{G}$-previsible process such that $H^f_i(\tau_i) = \Delta M^f_{\tau_i}$.

In practice, this last property is used to compute explicitly the "hedging ratios" H^f_i. For reason that will be clear soon, we will refer to this method of computing the H_i's as the *jump trick*.

PROOF. We know that the càdlàg version of the martingale M^f is given by

$$M^f_t = \sum_{\epsilon \in \{0,1\}^2} J_t(\epsilon)(\tau_1, \tau_2) E(\epsilon, \tau_1 \wedge t, \tau_2 \wedge t, f),$$

and from this equality one may deduce that

$$\begin{aligned} \Delta M^f_{\tau_1} = I_{\rrbracket 0,\tau_2 \rrbracket}(\tau_1)\big(E((0,1),\tau_1,\tau_1) - E((1,1),\tau_1,\tau_1)\big) \\ + I_{\rrbracket \tau_2,+\infty \llbracket}(\tau_1)\big(E((0,0),\tau_1,\tau_1) - E((1,0),\tau_1,\tau_1)\big), \end{aligned}$$

$$\begin{aligned} \Delta M^f_{\tau_2} = I_{\rrbracket 0,\tau_1 \rrbracket}(\tau_2)\big(E((1,0),\tau_2,\tau_2) - E((1,1),\tau_2,\tau_2)\big) \\ + I_{\rrbracket \tau_1,+\infty \llbracket}(\tau_2)\big(E((0,0),\tau_2,\tau_2) - E((0,1),\tau_2,\tau_2)\big). \end{aligned}$$

To understand these formulas, let's consider the case where $\tau_2 < \tau_1$. Taking a look at figure 1, it is very easy to convince oneself that on the event $\{\tau_2 < \tau_1\}$, one has

$$\Delta M^f_{\tau_1} = E(0,0)_{\tau_1,\tau_1} - E(1,0)_{\tau_1,\tau_1} \,, \quad \Delta M^f_{\tau_2,\tau_2} = E(1,0)_{\tau_2,\tau_2} - E(1,1)_{\tau_2,\tau_2} \,.$$

As the preceding equations may be rewritten as $\Delta M^f_{\tau_1} = H^f_1(\tau_1)$ and $\Delta M^f_{\tau_2} = H^f_2(\tau_2)$ where H^f_1 and H^f_1 are the previsible processes

$$\begin{aligned} H^f_1(t) &= I_{\rrbracket 0,\tau_2 \rrbracket}(t)\big(E(0,1)_{t,t} - E(1,1)_{t,t}\big) + I_{\rrbracket \tau_2,\infty \rrbracket}(t)\big(E(0,0)_{t,t} - E(1,0)_{t,t}\big), \\ H^f_2(t) &= I_{\rrbracket 0,\tau_1 \rrbracket}(t)\big(E(1,0)_{t,t} - E(1,1)_{t,t}\big) + I_{\rrbracket \tau_1,\infty \rrbracket}(t)\big(E(1,0)_{t,t} - E(1,1)_{t,t}\big), \end{aligned}$$

proposition 1 implies that formula (3) is true for the processes H^f_i we have just defined. ■

[6] The previsible σ-field is generated by the continuous $\mathcal{G}$-adapted process, see [R-W] or [Pro].

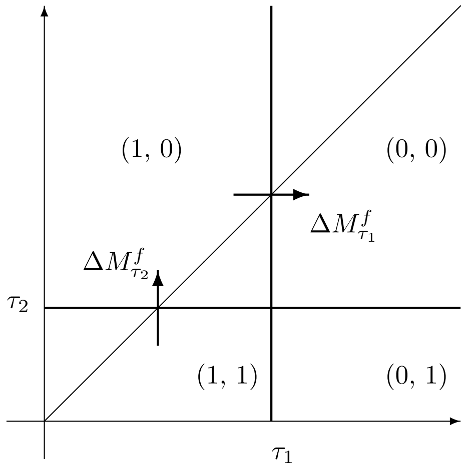

FIGURE 1: Determination of $\Delta M^f_{\tau_i}, i = 1, 2$ in the case $\tau_2 < \tau_1$.

3 Dynamic hedging

3.1 Replicating Portfolio

We now assume that the Credit Default Swap contracts are liquid and we consider a portfolio of the form

$$P_t = \gamma_1(t)CDS^1(t) + \gamma_2(t)CDS^2(t) + \gamma_3(t)cash - CD(t),$$

which is assumed to be self-financing in the sense that

$$\mathrm{d}P_t = \gamma_1(t)\,\mathrm{d}CDS^1(t) + \gamma_2(t)\,\mathrm{d}CDS^2(t) - \mathrm{d}CD(t).$$

We will also assume that the initial value of the portfolio is zero. We aim at finding a two-dimensionnal process γ such that the portfolio P is riskless. The representation results we recall in section 2 allows us to rewrite the preceding equation as

$$\mathrm{d}P = \big(\gamma_1 H^1_1 + \gamma_2 H^2_1 - K_1\big)\,\mathrm{d}M^1 + \big(\gamma_1 H^1_2 + \gamma_2 H^2_2 - K_2\big)\,\mathrm{d}M^2$$
$$= \gamma_1 H^1 \cdot \mathrm{d}M_t + \gamma_2 H^2 \cdot \mathrm{d}M - K \cdot \mathrm{d}M\,,$$

where H^i and K are the 2-dimensional previsible processes such that

$$CDS^i(t) = CDS^i(0) + \int_{]0,t]} H^i_s \cdot \mathrm{d}M_s\,, \quad (i = 1, 2),$$
$$CD(t) = CD(0) + \int_{]0,t]} K_s \cdot \mathrm{d}M_s\,,$$

and, using the *jump trick*, on may easily prove that, for all $t \le T$,

$$H^1_1(t) = I_{]\!]0,\tau_2]\!]}(t)\frac{G(T,t)}{G(t,t)} + I_{]\!]\tau_2,+\infty[\![}(t)\frac{(\partial_2 G)(T,t)}{(\partial_2 G)(t,t)}\,,$$
$$H^1_2(t) = I_{]\!]0,\tau_1]\!]}(t)\Big(\frac{G(T,t)}{G(t,t)} - \frac{(\partial_2 G)(T,t)}{(\partial_2 G)(t,t)}\Big),$$
$$H^2_1(t) = I_{]\!]0,\tau_2]\!]}(t)\Big(\frac{G(t,T)}{G(t,t)} - \frac{(\partial_1 G)(t,T)}{(\partial_1 G)(t,t)}\Big),$$
$$H^2_2(t) = I_{]\!]0,\tau_1]\!]}(t)\frac{G(t,T)}{G(t,t)} + I_{]\!]\tau_1,+\infty[\![}(t)\frac{(\partial_1 G)(t,T)}{(\partial_1 G)(t,t)}\,,$$

$$K_1(t) = I_{]\!]0,\tau_2]\!]}(t)\left(C_1\frac{G(T,T)}{G(t,t)} + C_2\Big[\frac{G(t,T)+G(T,t)-G(T,T)}{G(t,t)} - \frac{(\partial_1 G)(t,T)}{(\partial_1 G)(t,t)}\Big]\right) + C_2 I_{]\!]\tau_2,+\infty[\![}(t)\frac{(\partial_2 G)(T,t)}{(\partial_2 G)(t,t)}\,,$$

$$K_2(t) = I_{]\!]0,\tau_1]\!]}(t)\left(C_1\frac{G(T,T)}{G(t,t)} + C_2\Big[\frac{G(t,T)+G(T,t)-G(T,T)}{G(t,t)} - \frac{(\partial_2 G)(T,t)}{(\partial_2 G)(t,t)}\Big]\right) + C_2 I_{]\!]\tau_1,+\infty[\![}(t)\frac{(\partial_1 G)(t,T)}{(\partial_1 G)(t,T)}\,.$$

Now, we observe that our portfolio is riskless or insensitive to jump-risk[7] as soon as $\mathrm{d}P = 0$ and in view of the orthogonality of the martingales M^1 and M^2 this is equivalent to

$$\begin{cases} \gamma_1 H_1^1 + \gamma_2 H_1^2 = K_1 \\ \gamma_1 H_2^1 + \gamma_2 H_2^2 = K_2\,. \end{cases}$$

In other words, the hedging ratios $\gamma = (\gamma_1, \gamma_2)$ we are looking for are solution of the linear system $\mathbf{H}\gamma = K$, where $\mathbf{H} = (H_i^j)$. Following this hedging strategy, we then have for all $t \le T$

$$P_t = CD(0) + \int_{]0,t]} \gamma_1(s)\,\mathrm{d}CDS^1(s) + \int_{]0,t]} \gamma_2(s)\,\mathrm{d}CDS^2(s) - CD(t) = 0.$$

In other words,

$$CD(t) = CD(0) + \int_{]0,t]} \gamma_1(s)\,\mathrm{d}CDS^1(s) + \int_{]0,t]} \gamma_2(s)\,\mathrm{d}CDS^2(s).$$

3.2 Numerical Applications

In this section, we apply the preceding result to a Copula Model [SCH] on two reference entities F_1 and F_2. More precisely, we assume that the risk-neutral law of the default times (τ_1, τ_2) of the two considered firms is such that the marginals are exponentially distributed with rate λ_1 and λ_2 respectively and that the corresponding survival copula is the Clayton copula with parameter $\theta > 0$ (see for instance [JOE] or [RON]):

$$\tilde{C}(u, v; \theta) = \Big(\frac{1}{u^\theta} + \frac{1}{v^\theta} - 1\Big)^{-1/\theta}.$$

In other words, we will assume that

$$G(t, s) = \tilde{C}(\mathrm{e}^{-\lambda_1 t}, \mathrm{e}^{-\lambda_2 t}, \theta) = \big(\mathrm{e}^{\theta\lambda_1 t} + \mathrm{e}^{\theta\lambda_2 s} - 1\big)^{-1/\theta}.$$

We introduce the following notations: T maturity, N step number, $\mathrm{d}t = T/N$ discretization step, $t_k = k \times \mathrm{d}t$, $(0 \le k \le N)$ and we study the *tracking error* defined as

$$TE(t_k) = CD(0) + \sum_{i=0}^{k-1} \gamma(t_i)\cdot\big(CDS(t_{i+1}) - CDS(t_i)\big) - CD(t_k).$$

We draw some graph associated to the following values for the parameter:

$\lambda_1 = 700bp$,	$\lambda_2 = 500bp$,	$\theta = 10.0$
$T = 10Y$,	$Freq = 50$ step per year,	(weekly rebalanced portfolio)
$C_1 = 5\%$,	$C_2 = 20\%$.	$Notional = 1.0$

[7] we insist on the fact the only source of randomness here is τ.

On graph 1 and graph 2, we plot the value, as time goes by, of the CDSs, the Credit Derivatives, the Tracking Error and the Hedge Ratios. On this example there is two defaults: firms F_1 and F_2 defaulted about 3 and 7 years after inception respectively. After the default of the corresponding firm, the value of the CDS jumps to 1 which is coherent with formulas (1). In the same way, after the first default, the Credit Derivatives value experienced a positive jump and after the second default its value jumps to 25% accordingly to formula (2).

After the default of firm F_1 the value of Credit Default Swap on F_2 increase and this express the positive correlation between the two default time: after the first default the default probability of the remaining firm increases and so do the value of the corresponding CDS. We may also observe that with our choice of payoff our products shows negative θ.

After the default of the firm F_1 the corresponding hedging ratio disappear and so do the other ratio after the second default. We verify that the tracking error decreases as the frequency of rebalancing increases. We also plot on graph 3 the approximated loss distribution that occur when one follows this strategy. We observe that this distribution possess two modes: the negative one correspond to the case where there is no default and express the fact that the θ of the portfolio is negative; the positive one correspond to the case when there is at least one default and in this case our portfolio end with a positive value.

We also graph the frequency histogram for the tracking error (graph 3) and the frequency histogram in logarithmic scale (graph 4 and 5).

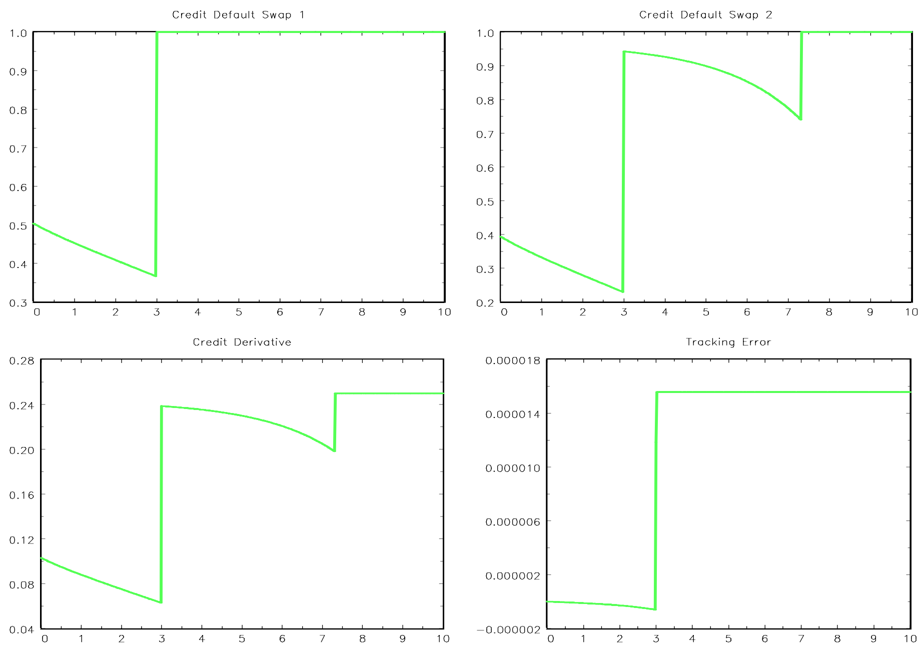

FIGURE 2: graph 1

3.3 What about the General Case?

In the general case, there is N firms (or obligors) in the basket and the situation is much more intricate but it is quite easy to convince oneself that the same reasoning may quite well be applied:

Firstly, one has to compute using *jump trick* the previsible coefficient $\mathbf{H} = (H^i_j)$ such

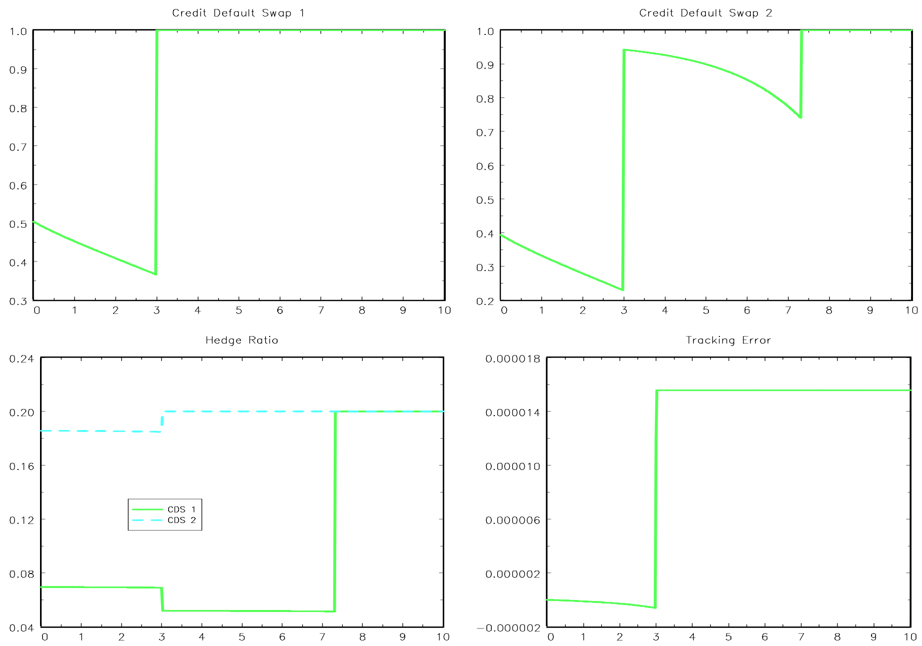

FIGURE 3: graph 2

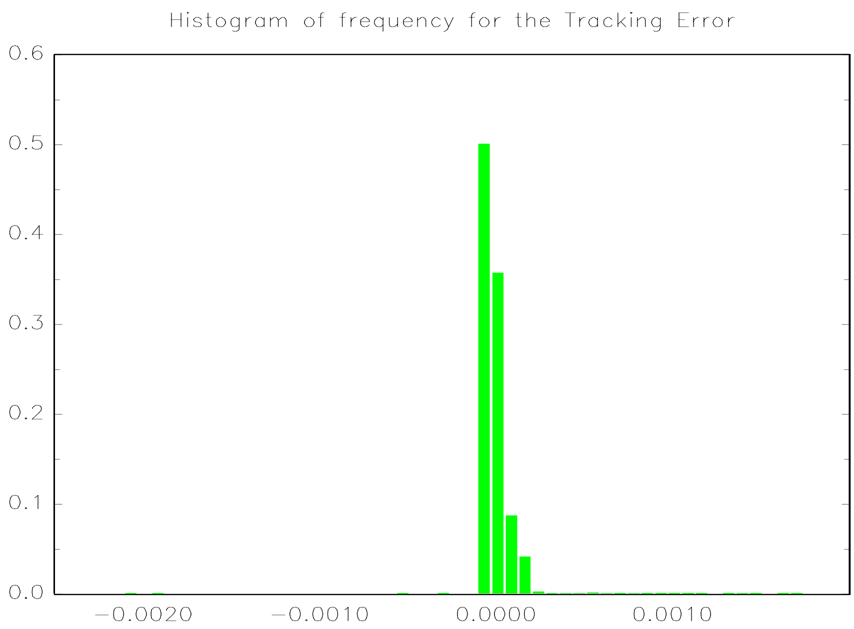

FIGURE 4: graph 3

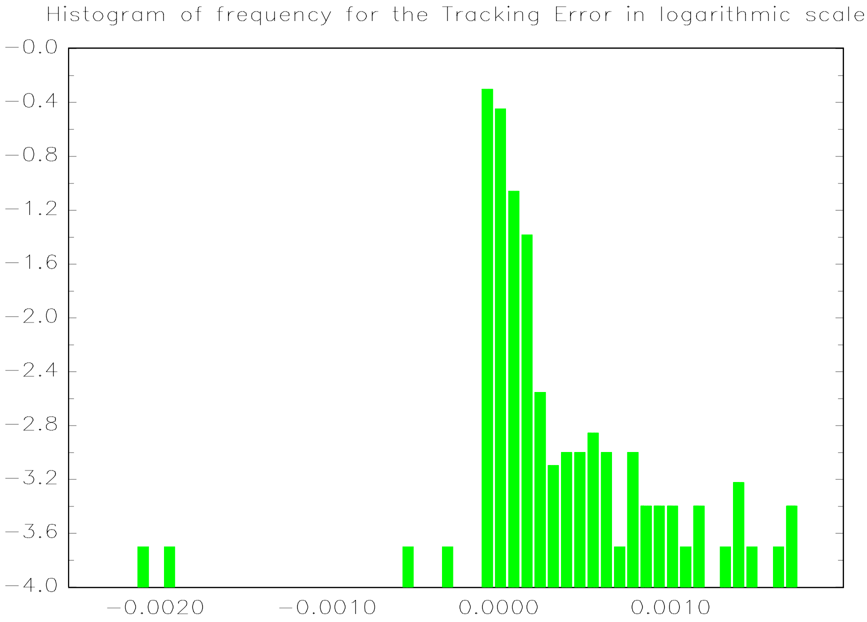

FIGURE 5: graph 4

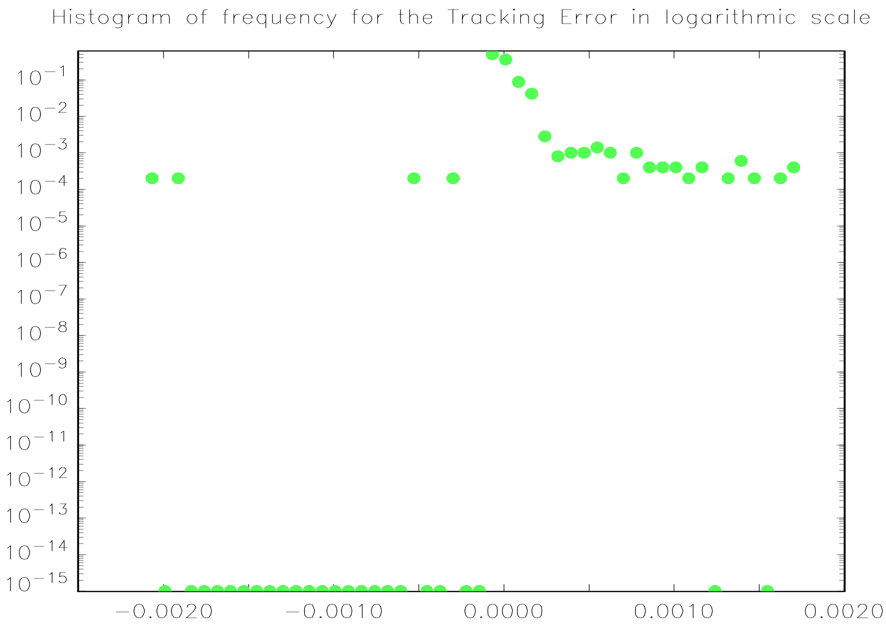

FIGURE 6: graph 5

that

$$\mathrm{d}CDS^i(t) = \sum_j H^i_j \,\mathrm{d}M^j.$$

Secondly, and considering a complex derivatives whose price process is P, one has to compute (*jump trick* again) the previsible coefficient (K_j) such that

$$\mathrm{d}P = \sum_j K_j \,\mathrm{d}M^j\,.$$

Then to find the hedging portfolio it suffices, in principle, to solve in γ the following system of linear equation: $\mathbf{H}\gamma = K$. Of course, to be perfectly rigorous one has to prove first that in this context this system admit a solution.

4 Conclusion

Recall that we are working in the framework of reduced-form model with deterministic interest rate and spread and that the only crucial hypothesis is that the survival function of the law of the default time is smooth. We present in this note some techniques that may be use to build a self-financing portfolio which replicates the payoff of a Basket Credit Derivatives. These techniques only use Non-arbitrage arguments and elementary stochastic calculus (with jumps).

More precisely, we prove that under some mild regularity assumptions the market described by our model is *complete* with respect to Credit Default Swap. In other words, under our working hypothesis, one could perfectly hedge any Credit Derivatives using Credit Default Swaps as basic underlying. Furthermore, we show that the corresponding hedging ratios may be easily computed by using what we call the *jump trick*); we choose this name to express the fact that these hedging ratios are completely described once one know the amplitude of the jump involved by the choosen model.

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