An Internal Model for Operational Risk Computation*

Seminarios de Matemática Financiera

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Agenda

1. What is operational risk ?
2. The New Basel Capital Accord
3. What is LDA?
4. Computing the Capital-at-Risk
5. Some pratical issues

1 What is operational risk?

An informal survey [...] highlights the growing realization of the significance of risks other than credit and market risks, such as **operational risk***, which have been at the heart of some important banking problems in recent years.* (Basel Committee on Banking Supervision, June 1999)

operational risk = financial risk other than credit and market risks.

Some examples of operational risk:

- Internal and external frauds
- Crédit Lyonnais headquarter fire (disasters)
- Barings (failure of control)

⇒ Operational Risk Losses Database of the BBA (British Bankers' Association)

2 The New Basel Capital Accord

The 1988 Capital Accord only concerns credit risk (and market risk — Amendment of January 1996) $\Rightarrow$ the Cooke Ratio requires capital to be at least 8 percent of the "risk" of the bank.

- January 2001: proposal for a New Basel Capital Accord (credit risk measurement will be more risk sensitive + explicit capital calculations for operational risk)
- August 2001: QIS 2 (Quantitative Impact Study)
- September 2001: Working Paper on the "Regulatory Treatment of Operational Risk"

$\Rightarrow$ The objectives of the New Accord are the following:

1. capital calculations will be more risk sensitive
2. convergence between economic capital and regulatory capital

The McDonough ratio

It is defined as follows:

$$\frac{\text{Capital (Tier I and Tier II)}}{\text{credit risk} + \text{market risk} + \text{operational risk}} \geq 8\%$$

The objective of allocation for the industry is not yet definitive:

Risk	January 2001	September 2001
Credit	75%	??
Market	5%	??
Operational	20%	12%

The definition of the Basel Committee

> **[...] the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.**

⇒ does not include systemic, strategic and reputational risks.

The loss type classification is the following:

1. **Internal Fraud**
2. **External Fraud**
3. **Employment Practices & Workplace Safety**
4. **Clients, Products & Business Practices**
5. **Damage to Physical Assets**
6. **Business Disruption & System Failures**
7. **Execution, Delivery & Process Management**

1. **Internal Fraud**: *losses due to acts of a type intended to defraud, misappropriate property or circumvent regulations, the law or company policy, excluding diversity/discrimination events, which involves at least one internal party*.
2. **External Fraud**: *losses due to acts of a type intended to defraud, misappropriate property or circumvent the law, by a third party*.
3. **Employment Practices & Workplace Safety**: *losses arising from acts inconsistent with employment, health or safety laws or agreements, from payment of personal injury claims, or from diversity/discrimination events*.
4. **Clients, Products & Business Practices**: *losses arising from an unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product*.
5. **Damage to Physical Assets**: *losses arising from loss or damage to physical assets from natural disaster or other events*.
6. **Business Disruption & System Failures**: *losses arising from disruption of business or system failures*.
7. **Execution, Delivery & Process Management**: *losses from failed transaction processing or process management, from relations with trade counterparties and vendors*.

The measurement methodologies

1. **Basic Indicator Approach (BIA)**
2. **Standardized Approach (SA)**
3. **Advanced Measurement Approach (AMA)**
    1. **Internal Measurement Approach (IMA)**
    2. **Loss Distribution Approach (LDA)**
    3. **Scorecard Approach (ScA)**

Basic Indicator Approach

$$K = \alpha \times \text{GI} \;\; \text{(Gross Income)}$$

Analysis of **QIS** data: $\alpha \simeq 20\%$ (based on 12% of minimum regulatory capital).

Qualifying criteria: none.

Standardized Approach The bank divides its activities into eight standardized business lines: **Corporate Finance**, **Trading & Sales**, **Retail Banking**, **Commercial Banking**, **Payment & Settlement**, **Agency Services & Custody**, **Retail Brokerage**, **Asset Management**.

$$K = \sum K_i = \sum \beta(i) \times \mathrm{EI}(i)$$

where EI is an exposure indicator for each of the 8 business lines.

Analysis of **QIS** data $(\mathrm{EI}(i) = \mathrm{GI}(i))$:

$\beta(i)$	Median	Mean	Minimum	Maximum
Corporate Finance	0.131	0.236	0.035	0.905
Trading & Sales	0.171	0.241	0.023	0.775
Retail Banking	0.125	0.127	0.008	0.342
Commercial Banking	0.132	0.169	0.048	0.507
Payment & Settlement	0.208	0.203	0.003	0.447
Agency Services & Custody	0.174	0.232	0.056	0.901
Retail Brokerage	0.113	0.149	0.050	0.283
Asset Management	0.133	0.185	0.033	0.659

Qualifying criteria: effective risk management, loss data, etc.

Advanced Measurement Approach The bank now divides its activities into the 8 business lines and the 7 risk types:

$$K = \sum \sum K(i,j)$$

- *Internal Measurement Approach* — $K(i,j) = \mathrm{EL}(i,j) \cdot \gamma(i,j)$ where EL is the expected loss (??) and γ is a scaling factor.
- *Scorecard Approach* — $K(i,j) = \mathrm{EI}(i,j) \cdot \omega(i,j) \cdot \mathrm{RS}(i,j)$ where EI is an exposure indicator, RS is a risk score and ω a scaling factor.
- *Loss Distribution Approach*

Remark 1 $\alpha = 99.9\%$

Remark 2 *Future of* **IMA** *and* **ScA** *?*

Remark 3 *A floor is set at 75% of the* **SA** *capital charge:*

$$K = \max\left(K_{\mathrm{AMA}}, \frac{3}{4} K_{\mathrm{SA}}\right)$$

3 What is LDA?

LDA = a statistical/actuarial approach for computing aggregate loss distributions (Klugman, Panjer and Willmot [1998]).

> *Under the Loss Distribution Approach, the bank estimates, for each business line/risk type cell, the probability distribution functions of the single event impact and the event frequency for the next (one) year using its internal data, and computes the probability distribution function of the cumulative operational loss* (Basel Committee on Banking Supervision, Operational Risk — Consultative Document, Supporting document to the New Basel Capital Accord, January 2001).

3.1 Analytic expression of the loss distribution

- We consider different business lines ($i = 1, \ldots, I$) and event types ($j = 1, \ldots, J$).
- $\zeta(i,j)$ is the random variable which represents the **amount of one loss event** for the business line i and the event type j. The **loss severity distribution** of $\zeta(i,j)$ is denoted by $\mathbf{F}_{i,j}$.
- We assume that the number of events between times t and $t+\tau$ is random. The corresponding variable $N(i,j)$ has a probability function $p_{i,j}$. The **loss frequency distribution** $\mathbf{P}_{i,j}$ corresponds to

$$\mathbf{P}_{i,j}(n) = \sum_{k=0}^{n} p_{i,j}(k)$$

In **LDA**, the loss for the business line i and the event type j between times t and $t+\tau$ is

$$\vartheta\left(i,j\right)=\sum_{n=0}^{N(i,j)}\zeta_{n}\left(i,j\right)$$

Let $\mathbf{G}_{i,j}$ be the distribution of $\vartheta\left(i,j\right)$. $\mathbf{G}_{i,j}$ is then a compound distribution:

$$\mathbf{G}_{i,j}\left(x\right)=\begin{cases}\sum_{n=1}^{\infty}p_{i,j}\left(n\right)\mathbf{F}_{i,j}^{n\star}\left(x\right) & x>0\\ p_{i,j}\left(0\right) & x=0\end{cases}$$

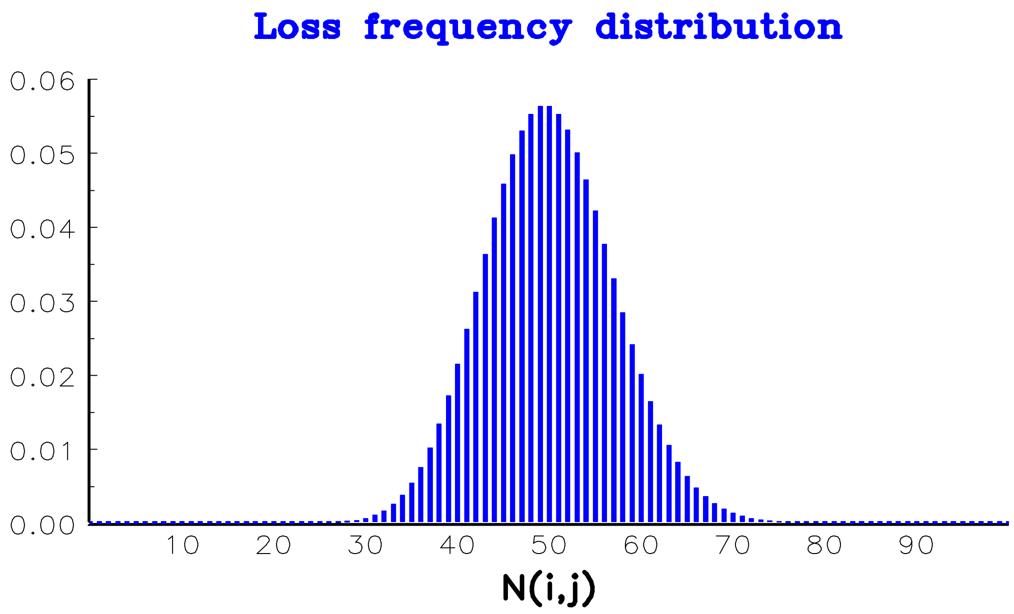
where $\star$ is the *convolution* operator on distribution functions and $\mathbf{F}^{n\star}$ is the n-fold convolution of $\mathbf{F}$ with itself.

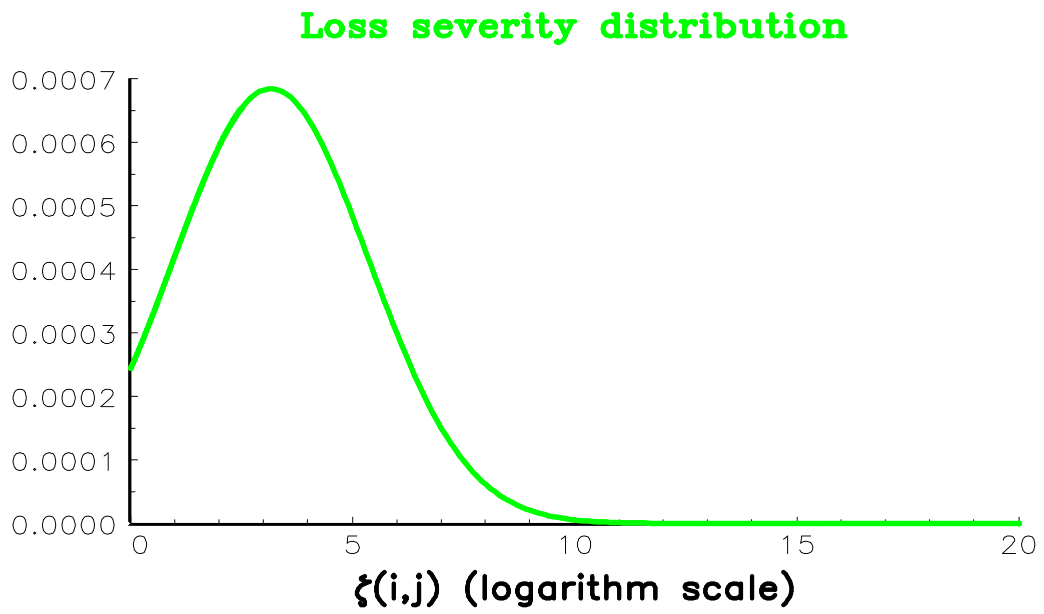
3.2 Computing the loss distribution function

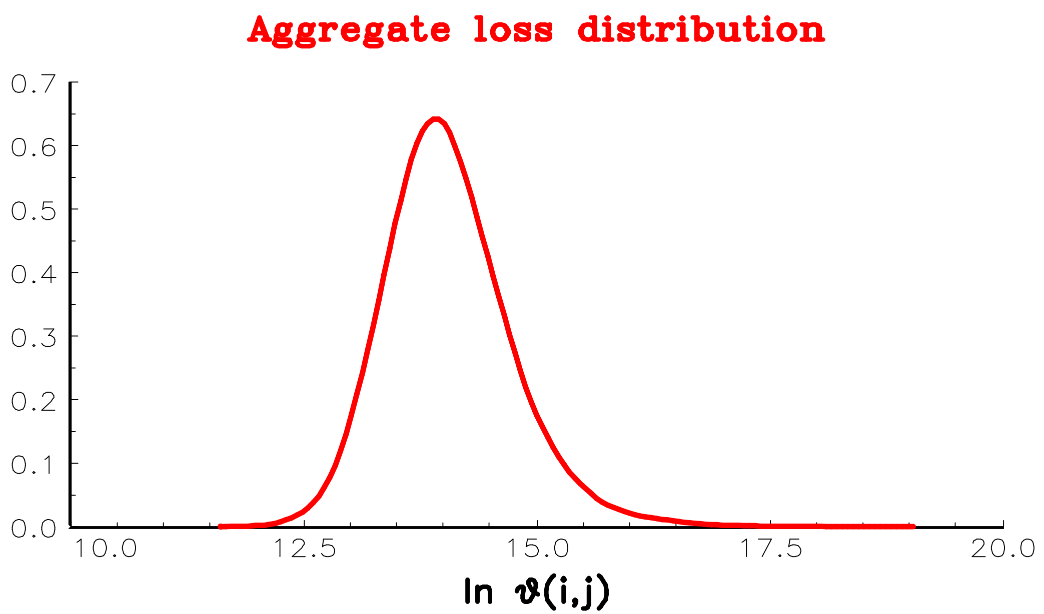
In general, no closed-formula for the probability distribution $\mathbf{G}_{i,j}$.

Numerical methods

- Monte Carlo method
- Panjer's recursive approach
- Inverse of the characteristic function







Computing the aggregate loss distribution with $N(i,j) \sim P(50)$ and $\xi(i,j) \sim LN(8,2.2)$

4 Computing the Capital-at-Risk

Capital-at-Risk for operational risk = *Value-at-Risk* measure (*Expected Loss* + *Unexpected Loss*):

$$\begin{aligned}\mathrm{CaR}\,(i,j;\alpha) &= \mathrm{EL}\,(i,j) + \mathrm{UL}\,(i,j;\alpha) \\ &= \mathbf{G}_{i,j}^{-1}\,(\alpha) \\ &:= \inf\left\{x \mid \mathbf{G}_{i,j}\,(x) \geq \alpha\right\}\end{aligned}$$

4.1 For one business line and one event type

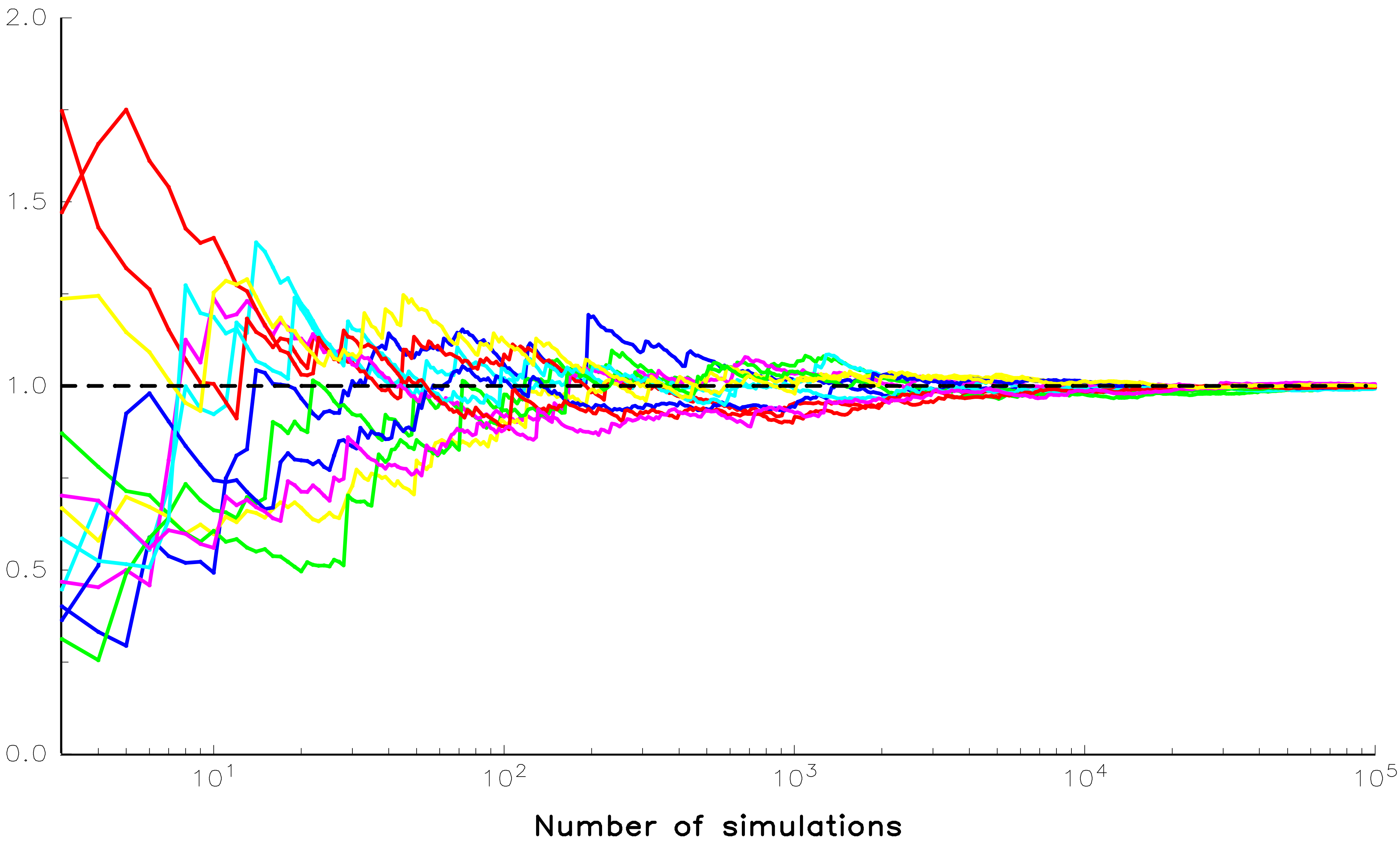
Problems of the quantile estimation:

1. the confidence level is high ($\alpha = 99.9\%$);
2. the support of the distribution is very large.

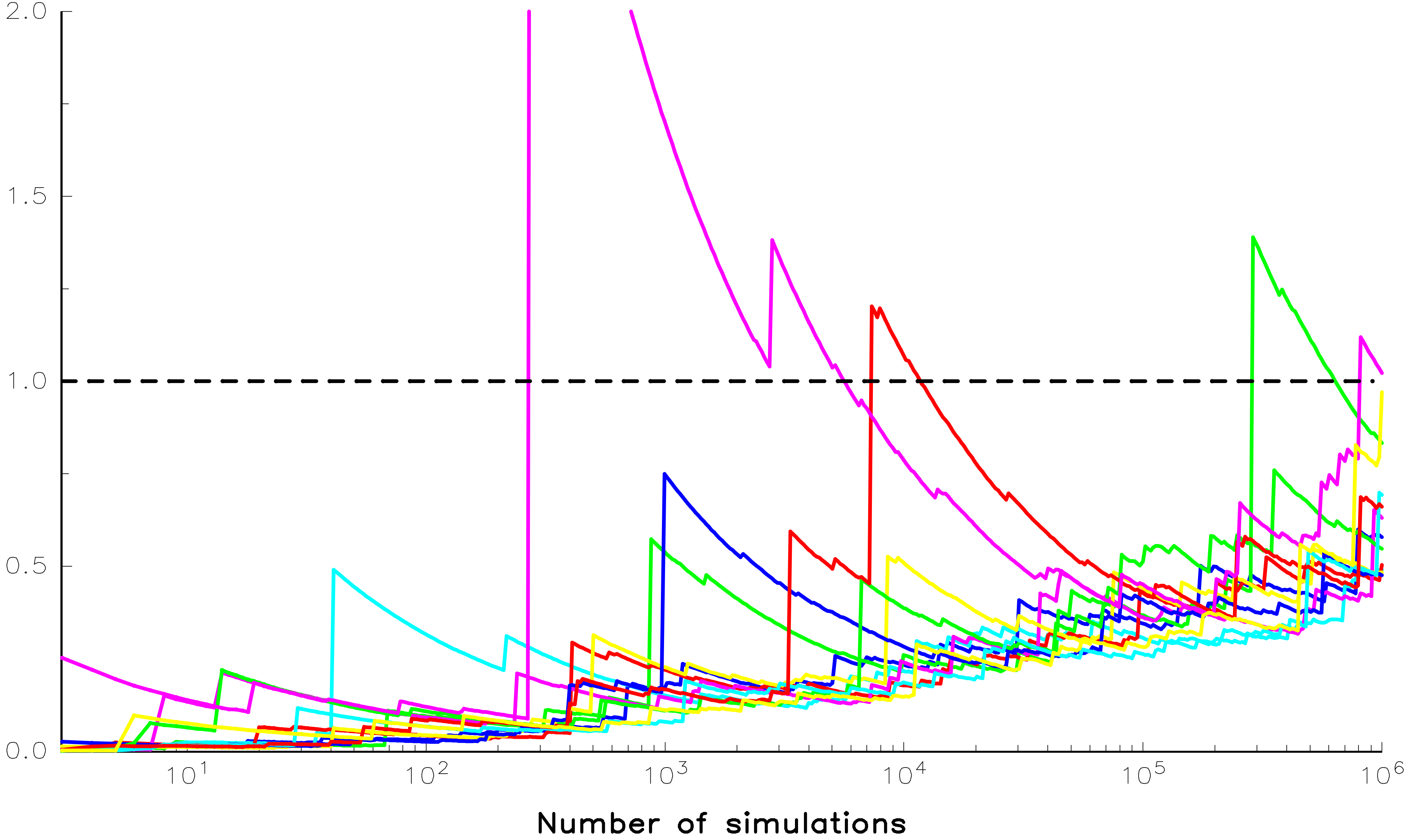
How to control the accuracy of the estimate $\hat{\mathbf{G}}_{i,j}^{-1}(\alpha)$?

$\Rightarrow$ Check the convergence of the first four moments and Large Deviations principles (Willmot and Lin [2000]):

$$
\begin{aligned}
\acute{\mu}_1^{\vartheta} &= \acute{\mu}_1^{N}\acute{\mu}_1^{\zeta} \\
\acute{\mu}_2^{\vartheta} &= \acute{\mu}_1^{N}\acute{\mu}_2^{\zeta} + \left(\acute{\mu}_2^{N} - \acute{\mu}_1^{N}\right)\left[\acute{\mu}_1^{\zeta}\right]^2 \\
\acute{\mu}_3^{\vartheta} &= \acute{\mu}_1^{N}\acute{\mu}_3^{\zeta} + 3\left(\acute{\mu}_2^{N} - \acute{\mu}_1^{N}\right)\acute{\mu}_1^{\zeta}\acute{\mu}_2^{\zeta} + \left(\acute{\mu}_3^{N} - 3\acute{\mu}_2^{N} + 2\acute{\mu}_1^{N}\right)\left[\acute{\mu}_1^{\zeta}\right]^3 \\
\acute{\mu}_4^{\vartheta} &= \acute{\mu}_1^{N}\acute{\mu}_4^{\zeta} + 4\left(\acute{\mu}_2^{N} - \acute{\mu}_1^{N}\right)\acute{\mu}_1^{\zeta}\acute{\mu}_3^{\zeta} + 3\left(\acute{\mu}_2^{N} - \acute{\mu}_1^{N}\right)\left[\acute{\mu}_2^{\zeta}\right]^2 + \ldots
\end{aligned}
$$



Convergence of the second moment
with N(i,j)~P(10) and ξ(i,j)~LN(5,1)



Convergence of the second moment
with N(i,j)~P(10) and ξ(i,j)~LN(5,3)

4.2 For the bank as a whole

Let ϑ be the total loss of the bank:

$$\vartheta = \sum_{i=1}^{I} \sum_{j=1}^{J} \vartheta(i,j)$$

In order to compute the aggregate loss distribution $\mathbf{G}$ or the total capital charge of the bank $\text{CaR}(\alpha) = \mathbf{G}^{-1}(\alpha)$, we **must** do some assumptions on the dependence function (**copula**) between the random variables $\vartheta(i,j)$.

Copulas in a nutshell A copula function $\mathbf{C}$ is a multivariate probability distribution with uniform $[0,1]$ margins.

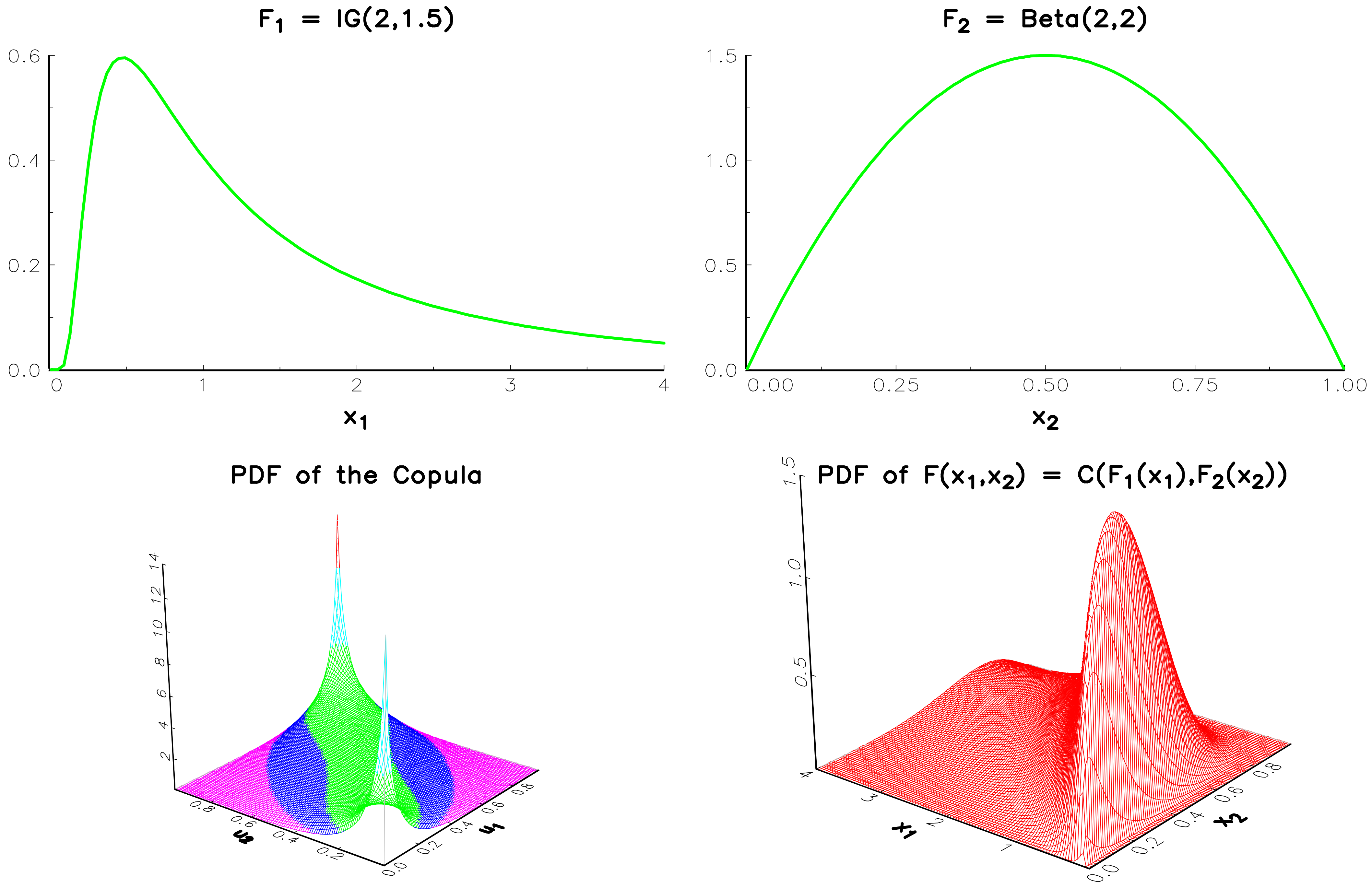
$\mathbf{C}(\mathbf{F}_1(x_1),\ldots,\mathbf{F}_N(x_N))$ defines a multivariate cdf $\mathbf{F}$ with margins $\mathbf{F}_1,\ldots,\mathbf{F}_N \Rightarrow \mathbf{F}$ is a probability distribution with given marginals (Fréchet classes).

The copula function of the random variables $(X_1,\ldots,X_N)$ is **invariant** under strictly increasing transformations:

$$\mathbf{C}\langle X_1,\ldots,X_N\rangle = \mathbf{C}\langle h_1(X_1),\ldots,h_N(X_N)\rangle$$

... the copula is invariant while the margins may be changed at will, it follows that is precisely the copula which captures those properties of the joint distribution which are invariant under a.s. strickly increasing transformations (Schweizer and Wolff [1981]).

$\Rightarrow$ **Copula = dependence function of r.v.** (Deheuvels [1978]).



Bivariate distribution with given marginals

Examples of copula function

- Normal copula

$$\mathbf{C}(u_1,\ldots,u_N;\rho) = \Phi\left(\Phi^{-1}(u_1),\ldots,\Phi^{-1}(u_N);\rho\right)$$

- Product copula

$$\mathbf{C}(u_1,\ldots,u_N) = u_1 \cdots \times u_N$$

  In this case, the random variables are independent.

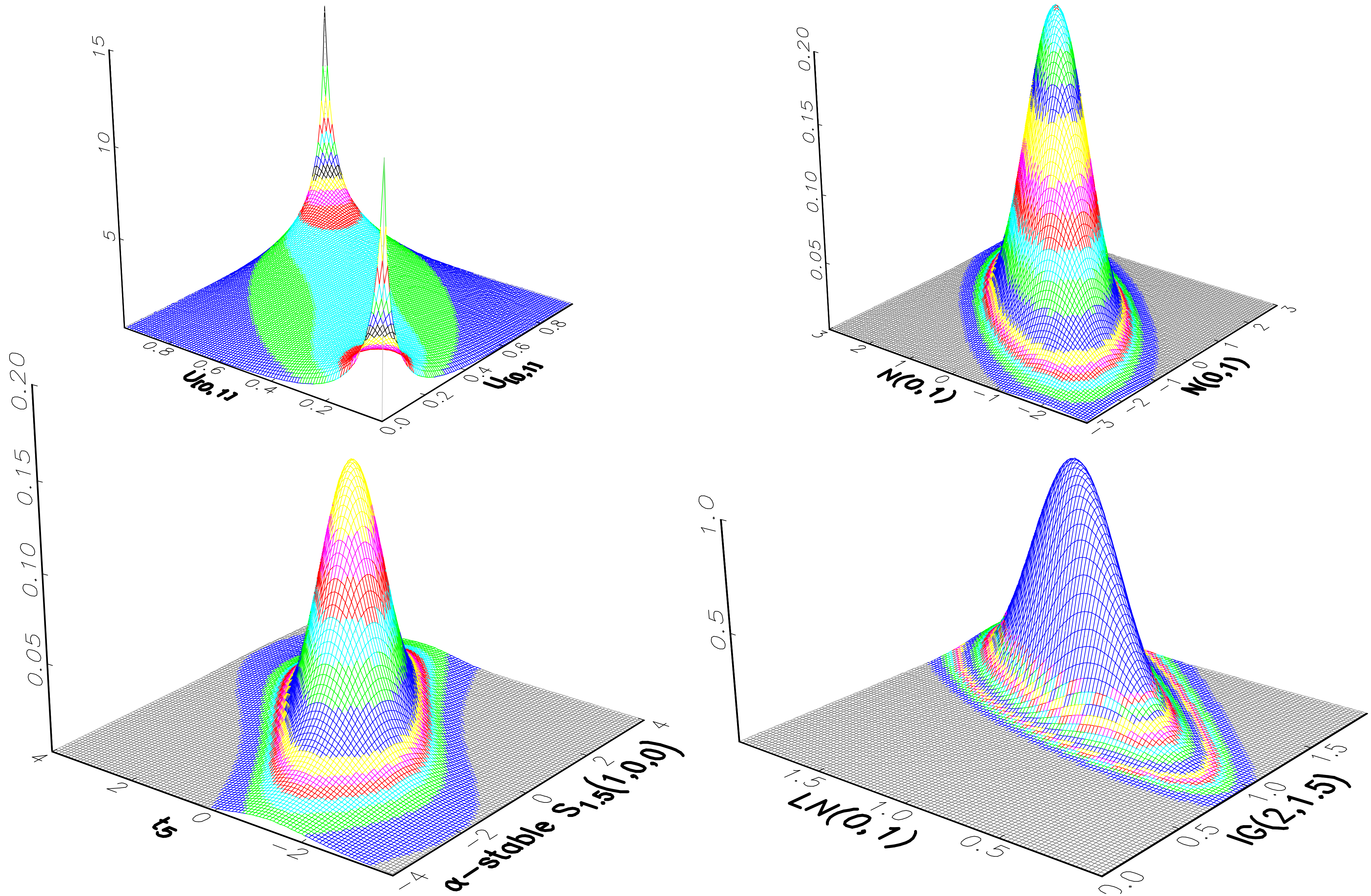
- Upper Fréchet copula

$$\mathbf{C}(u_1,\ldots,u_N) = \min(u_1,\ldots,u_N)$$

  In this case, the random variables are perfectly dependent. For example, we have

$$\Phi(x_1,\ldots,x_N;\mathbf{1}) = \min(\Phi(x_1),\ldots,\Phi(x_N))$$

- Other copulas: extreme value copulas, markov copulas, parametric copulas, non-parametric copulas, Deheuvels (or empirical) copulas.



Normal Copula with τ = 0.5

Canonical decomposition of the loss random variables

Let $\breve{\vartheta}$ be the vec form of the random variables $\vartheta(i,j)$:

$$\breve{\vartheta} = \text{vec}\left[\left(\vartheta(i,j)\right)_{i,j}\right]$$

We note $\breve{\mathbf{G}}$ the distribution of the random vector $\breve{\vartheta}$. By definition, the margins of $\breve{\mathbf{G}}$ are the individual aggregate loss distributions $\mathbf{G}_{i,j}$. Let $\mathbf{C}_{\left\langle \breve{\mathbf{G}} \right\rangle}$ be the copula function of $\breve{\vartheta}$. We have

$$\breve{\mathbf{G}}\left(x_{1,1},\ldots,x_{I,J}\right) = \mathbf{C}_{\left\langle \breve{\mathbf{G}} \right\rangle}\left(\mathbf{G}_{1,1}\left(x_{1,1}\right),\ldots,\mathbf{G}_{I,J}\left(x_{I,J}\right)\right)$$

$\Rightarrow$ In this case, we can define ϑ and CaR(α).

Some special cases

- If $\mathbf{C}_{\left\langle \breve{\mathbf{G}} \right\rangle} = \mathbf{C}^{+}$, then the total Capital-at-Risk is the sum of all CaRs:

$$\mathrm{CaR}\,(\alpha) = \sum_{i=1}^{I} \sum_{j=1}^{J} \mathrm{CaR}\,(i, j; \alpha)$$

  This is the method given by the Basel Committee on Banking Supervision in the Internal Measurement Approach. It corresponds to the assumption that the different risks are *totally positive dependent*, or roughly speaking, "perfectly correlated".

Proof (in the bivariate case)

Because $\mathbf{C}_{\langle \breve{\mathbf{G}} \rangle} = \mathbf{C}^{+}$, $\vartheta_2 = \mathbf{G}_2^{(-1)}\left(\mathbf{G}_1\left(\vartheta_1\right)\right)$. Let us denote ϖ the function $x \mapsto x + \mathbf{G}_2^{(-1)}\left(\mathbf{G}_1\left(x\right)\right)$. We have

$$\begin{aligned} \alpha &= \Pr\left\{\vartheta_1 + \vartheta_2 \leq \mathrm{CaR}\left(\alpha\right)\right\} \\ &= \mathbb{E}\left[\mathbf{1}_{\left[\varpi(\vartheta_1) \leq \mathrm{CaR}(\alpha)\right]}\right] \\ &= \mathbf{G}_1\left(\varpi^{-1}\left(\mathrm{CaR}\left(\alpha\right)\right)\right) \end{aligned}$$

It comes that $\mathrm{CaR}\left(\alpha\right) = \varpi\left(\mathbf{G}_1^{(-1)}\left(\alpha\right)\right)$ and we obtain the relationship

$$\mathrm{CaR}\left(\alpha\right) = \mathbf{G}_1^{(-1)}\left(\alpha\right) + \mathbf{G}_2^{(-1)}\left(\mathbf{G}_1\left(\mathbf{G}_1^{(-1)}\left(\alpha\right)\right)\right) = \mathrm{CaR}_1\left(\alpha\right) + \mathrm{CaR}_2\left(\alpha\right)$$

$\Rightarrow$ This relationship holds for all types of random variables, **not only for the gaussian case**.

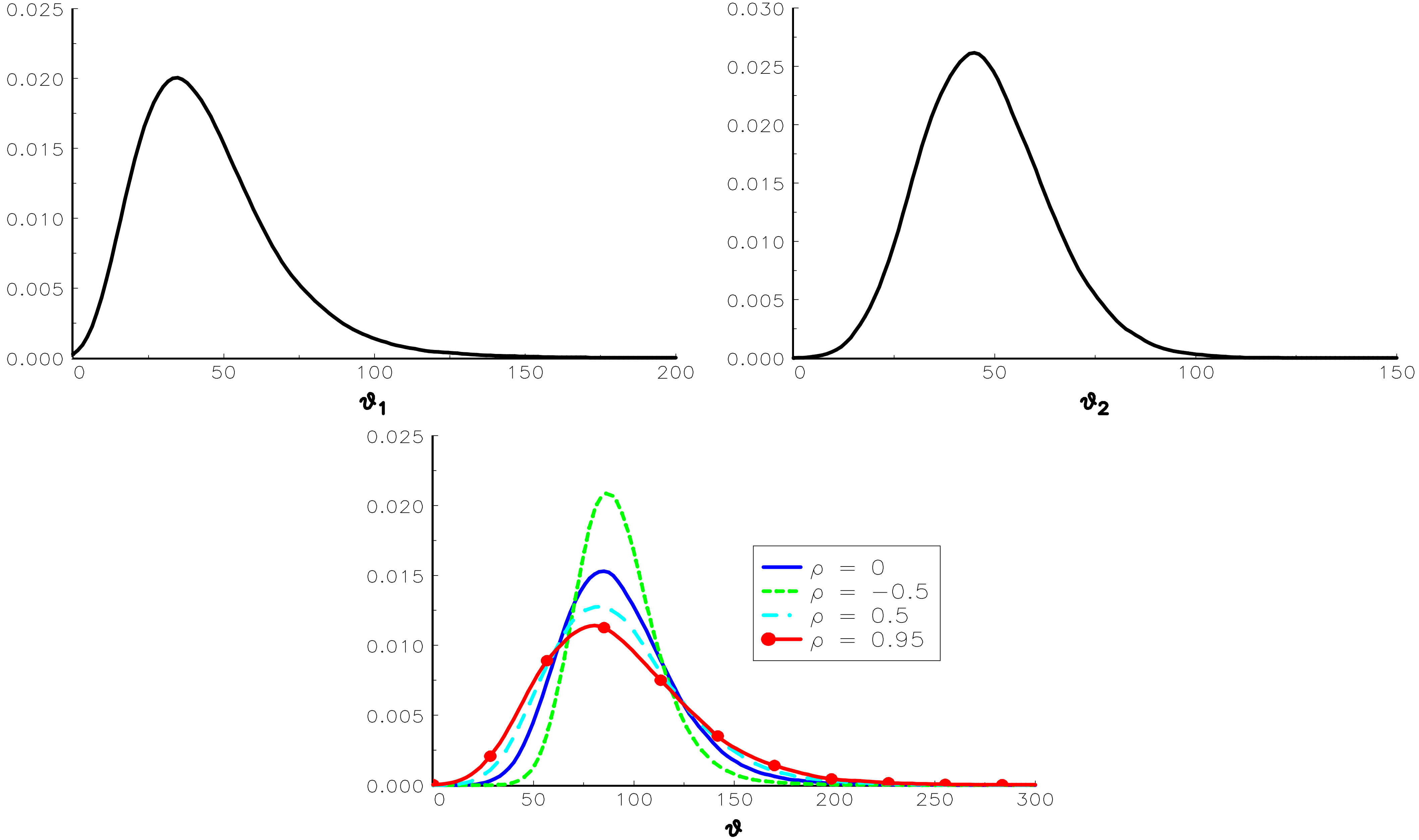
- If $\mathbf{C}_{\left\langle \breve{\mathbf{G}} \right\rangle} = \mathbf{C}^{\perp}$, then the total loss distribution is the convolution of the distributions $\mathbf{G}_{i,j}$:

$$\mathbf{G}(x) = \mathop{\star}_{i=1}^{I} \mathop{\star}_{j=1}^{J} \mathbf{G}_{i,j}(x)$$

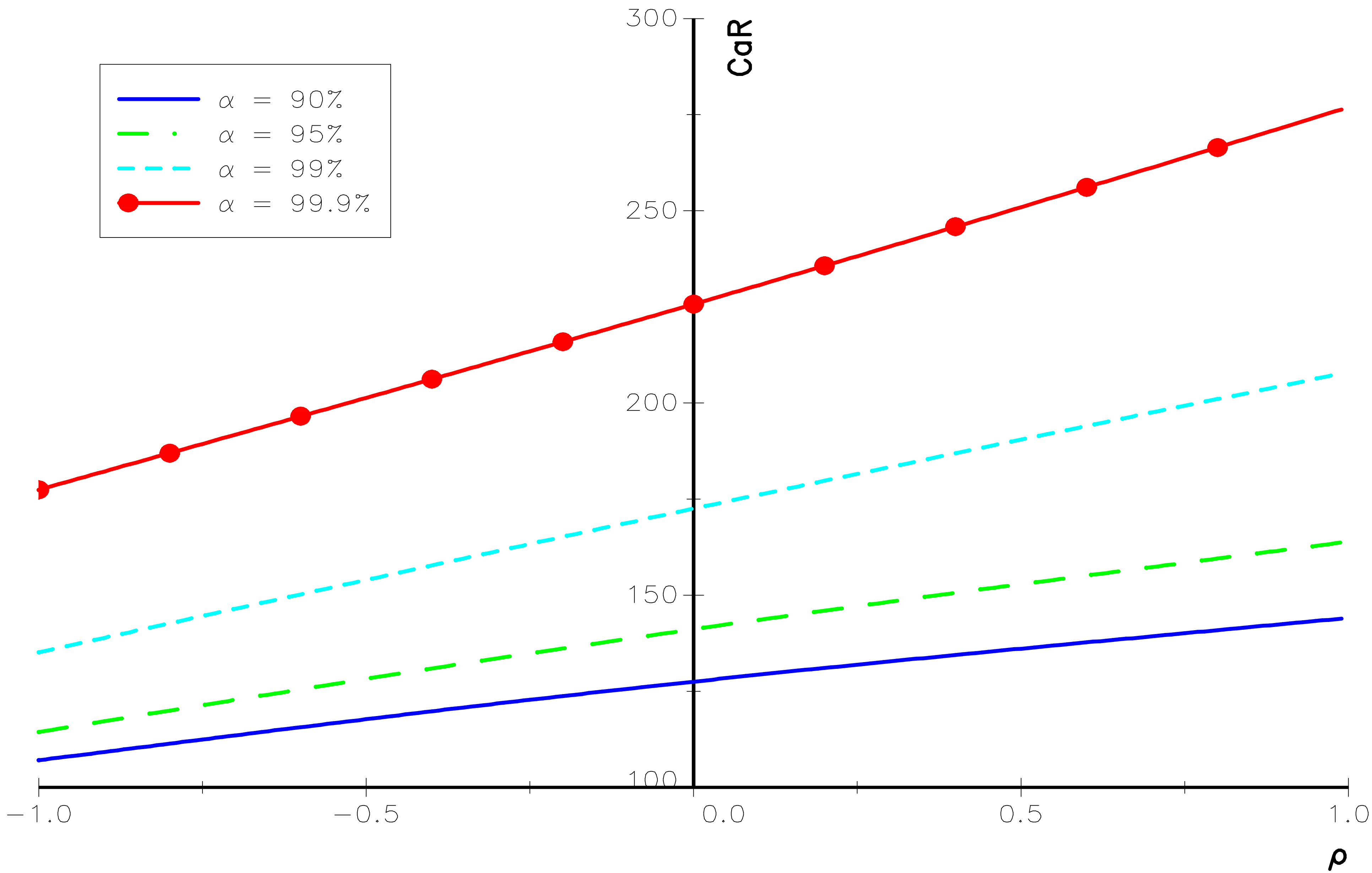
There is no analytical expression of the quantile function. Using a Normal approximation, we have

$$\mathrm{CaR}(\alpha) \simeq \sum_{i=1}^{I} \sum_{j=1}^{J} \mathrm{EL}(i,j) + \sqrt{\sum_{i=1}^{I} \sum_{j=1}^{J} \left[\mathrm{CaR}(i,j;\alpha) - \mathrm{EL}(i,j)\right]^2}$$

- In the other cases, $\mathrm{CaR}(\alpha)$ is computed thanks to the "empirical" Monte Carlo method.



Impact of the dependence function (Normal copula) on the total loss distribution



Impact of the dependence function (Normal copula)
on the Capital-at-Risk

Example with Crédit Lyonnais loss data

Some figures

Threshold: 1000 euro (losses bigger than this threshold **must** be reported) – Number of loss events ($\geq$ 1500 euro): $\simeq$ 6000 – "exhaustive" loss data for years 2000 and 2001.

$\Rightarrow$ Some loss types constitute the greater part of operational risk:

- Two loss types represent 67% of the capital charge (without diversification effect and risk mitigation).
- Two loss types represent 74% of the capital charge (with diversification effect – $\mathbf{C}_{\left\langle \breve{\mathbf{G}} \right\rangle} = \mathbf{C}^{\perp}$ – and risk mitigation).

Diversification effect We define the diversification ratio as follows

$$\chi(\alpha) = \frac{\mathrm{CaR}^{+}(\alpha) - \mathrm{CaR}(\alpha)}{\mathrm{CaR}^{+}(\alpha)}$$

where $\mathrm{CaR}^{+}(\alpha)$ is the *Capital-at-Risk* with $\mathbf{C}_{\langle \breve{\mathbf{G}} \rangle} = \mathbf{C}^{+}$. With $\mathbf{C}_{\langle \breve{\mathbf{G}} \rangle} = \mathbf{C}^{\perp}$, we obtain the following results:

Year	2000	2001
$\chi(99.9\%)$	33.2%	36.6%

Comparison Year 2000/Year 2001 (same frequencies)

- Without diversification effect: +5.5%
- With diversification effect: −7.7%
- 5 loss types for year 2000 ⇒ 7 loss types for year 2001 (Basle Committee classification)

5 Some pratical issues

Computing capital charge for operational risk

is not only a statistical problem,

but requires experience and expertise.

5.1 The data

Data is a crucial issue in operational risk management.

It requires an operational risk team and an effective organization to manage the data collection process.

Without (enough) loss data, calculation of capital charge can not be done.

Another question:

Internal data are sufficient to provide accurate capital charge?

or

Internal data should be supplemented with external data ?

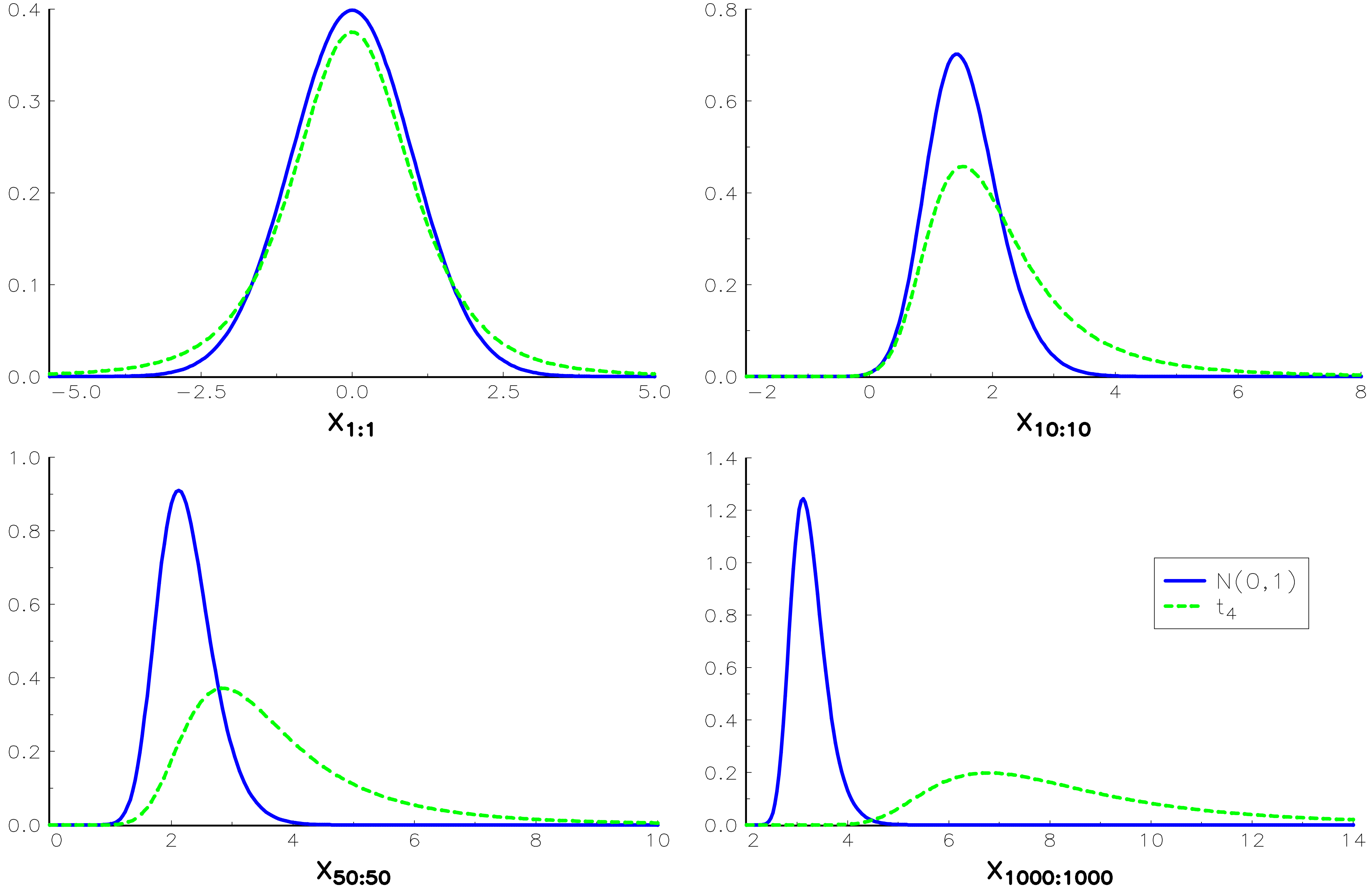
5.2 Selection criteria for the severity loss distribution

Non-parametric adequacy tests of distributions are not the most appropriate for operational risk.
$\Rightarrow$ the problem is not to fit the entire distribution, but the tail of the distribution.

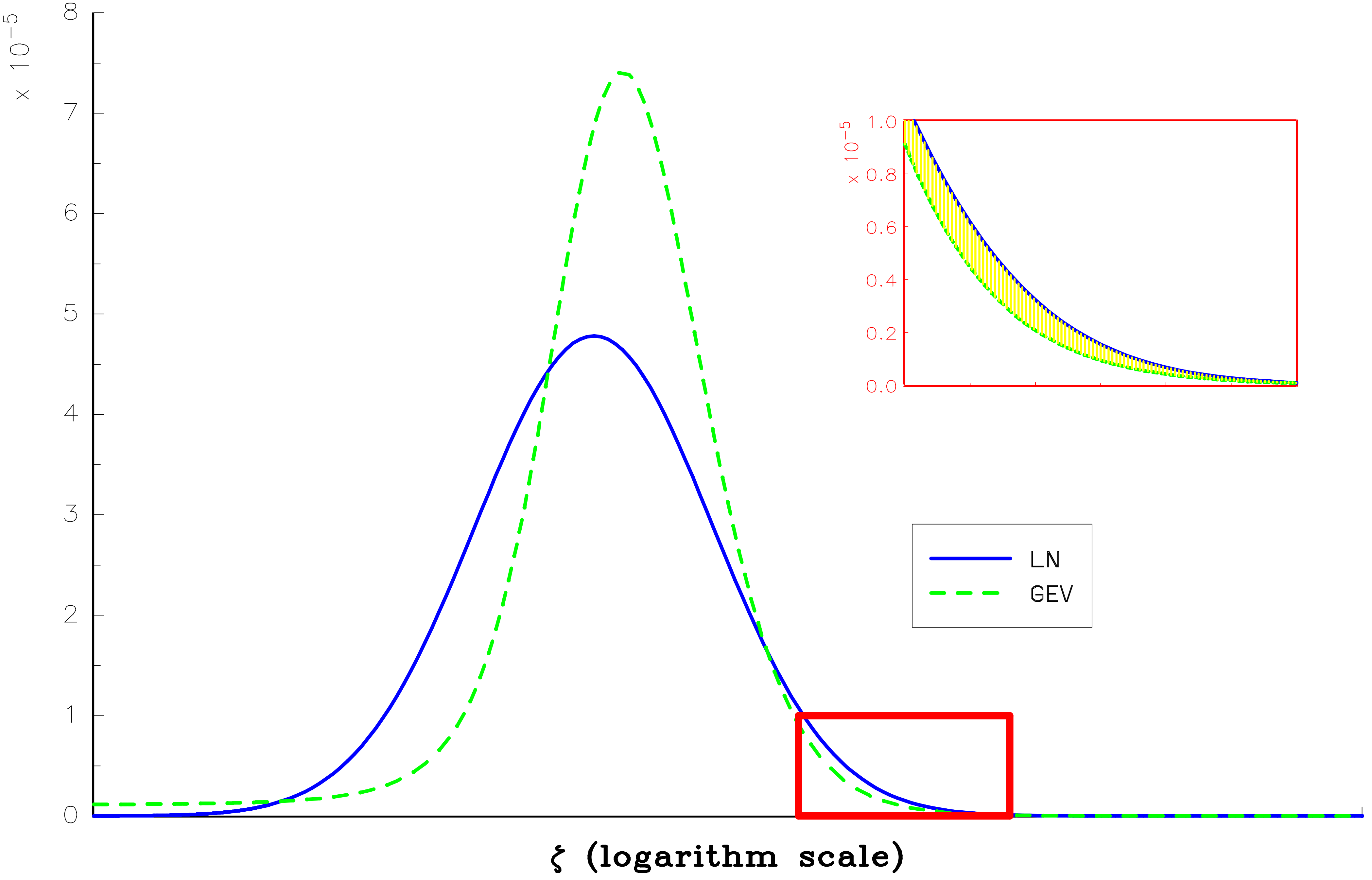
Selection criteria based on (extreme) order statistics is more appropriate. Let $X_1, \ldots, X_n$ be *i.i.d.* $\mathbf{H}$-distributed random variables. We define $X_{m:n}$ as the mth-order statistic in a sample size n. Then we have

$$X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{m:n} \leq \cdots \leq X_{n:n}$$

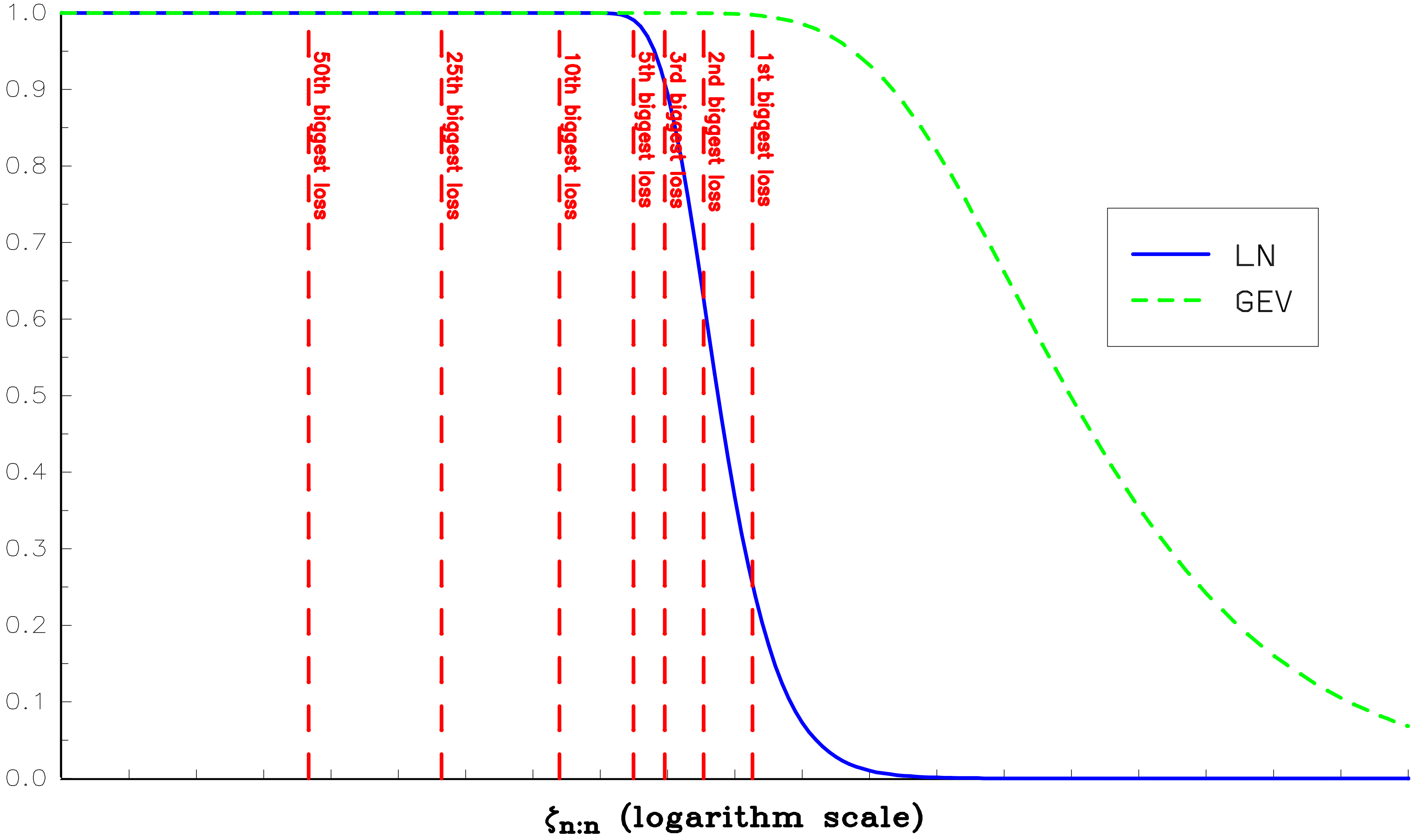
The distribution of the maximum $X_{n:n} = \max\left(X_1, \ldots, X_n\right)$
We have $\mathbf{H}_{n:n}\left(x\right) = \left[\mathbf{H}\left(x\right)\right]^n$ and $h_{n:n}\left(x\right) = n\left[\mathbf{H}\left(x\right)\right]^{n-1} h\left(x\right)$



Impact of the tails on the largest order statistic



Estimated LN and GEV distributions for the event type Fraud



Survival function of the largest order statistic
and observed biggest losses (Fraud)

5.3 Estimation methods to deal with aggregated data

Remark 4 *When an operational risk dataset is first created, it is not always possible to have a collect of all the past individual events.*

$\Rightarrow$ The dataset contains both individual losses and (few) aggregated losses.

Problem: it may be difficult to find the analytical expression of the distribution of aggregated losses.
$\Rightarrow$ Indirect inference, SMM, GMM, etc.

Example with GMM and $\mathcal{LN}$ distribution:

$$\left\{\begin{array}{rcl} h_{t,1}(\mu,\sigma) & = & \xi_t - n_t e^{\mu+\frac{1}{2}\sigma^2} \\ h_{t,2}(\mu,\sigma) & = & \left(\xi_t - n_t e^{\mu+\frac{1}{2}\sigma^2}\right)^2 - n_t e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right) \end{array}\right.$$

where ξ_t is the amount of aggregated loss for the observation t and n_t is the number of events corresponding to this observation.

5.4 Mixing internal and external data

Computing the expected frequency of events

Using credibility theory and under some assumptions, the expected number of events that relates best to the actual riskiness of the bank is a weighted linear combination:

$$\lambda = \varpi \times \lambda_{\text{Industry/External}} + (1 - \omega) \times \lambda_{\text{Bank/Internal}}$$

Mixing internal and external severity data

Internal databases should be supplemented with external data in order to give a non-zero likelihood to rare events which *could be missing* **in internal databases. However mixing internal and external data altogether may provide unacceptable results as external databases are strongly biased toward high-severity events.**

The framework

$f(\zeta;\theta)$: true loss probability density function where θ is a set of unknown parameters.

- two types of data:

- internal data
- external data

$\Rightarrow$ Internal and external data follow the same distribution, but external data are truncated by a (unknown and high) threshold H.

Remark 5 *Banks generally report big losses. External data are also biased toward high severity events as only large losses are publicly released.*

The statistical problem

ζ_j (respectively ζ_j^*) denotes an internal (resp. external) single loss record. Then, MLE is defined by

$$
\begin{aligned}
\left(\hat{\theta}, \hat{H}\right) &= \arg\max \sum_{j \in \mathcal{J}} \ln f\left(\zeta_j; \theta\right) + \sum_{j \in \mathcal{J}^*} \ln f_{|H}\left(\zeta_j^*; \theta\right) \\
&= \arg\max \sum_{j \in \mathcal{J}} \ln f\left(\zeta_j; \theta\right) + \sum_{j \in \mathcal{J}^*} \ln f\left(\zeta_j^*; \theta\right) - n^* \ln \int_H^{+\infty} f(\zeta; \theta)\, \mathrm{d}\zeta
\end{aligned}
$$

where n^* is the number of external losses.

$\Rightarrow$ H is unknown. It may be also not the same for all banks. In this case, we may assume that H is a random variable.

An example to show that a statistical method which ignores truncation is totally misleading

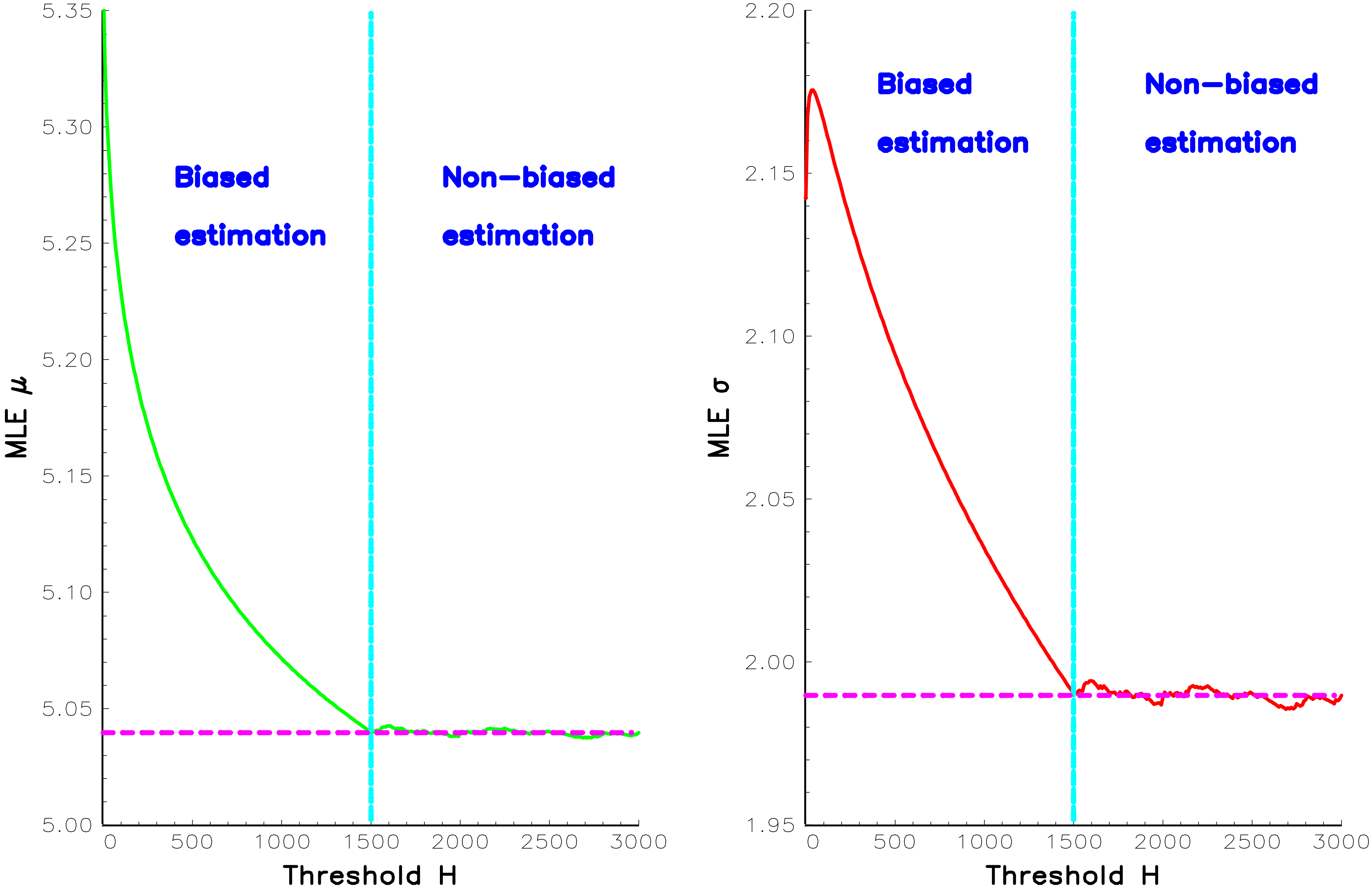
We assume that $\zeta \sim \mathcal{LN}(5, 2)$ and $H = 1500$. We simulate 2000 random variates from this distribution and we suppose that they are the internal data. Then, we simulate 2000 other random variates. We suppose that variates which take a value above the threshold represent the external data. In our simulation, $n^* = 219$.

The log-likelihood function is

$$\begin{aligned}\ell(\mu, \sigma, H) \;\propto\; & -(n + n^*)\ln\sigma + \sum_{j \in \mathcal{J}} \ln\phi\left(\frac{\ln\zeta_j - \mu}{\sigma}\right) + \\ & \sum_{j \in \mathcal{J}^*} \ln\phi\left(\frac{\ln\zeta_j^* - \mu}{\sigma}\right) - n^* \ln\left(1 - \Phi\left(\frac{\ln\zeta_j^* - \mu}{\sigma}\right)\right)\end{aligned}$$

$\Rightarrow$ If we mix directly internal and external data ($H = 0$), ML estimates are biased.

$\Rightarrow$ We remark that ML estimates are unbiased for $H \geq 1500$.



Impact of the threshold on the ML estimates

An example with Crédit Lyonnais data and BBA data

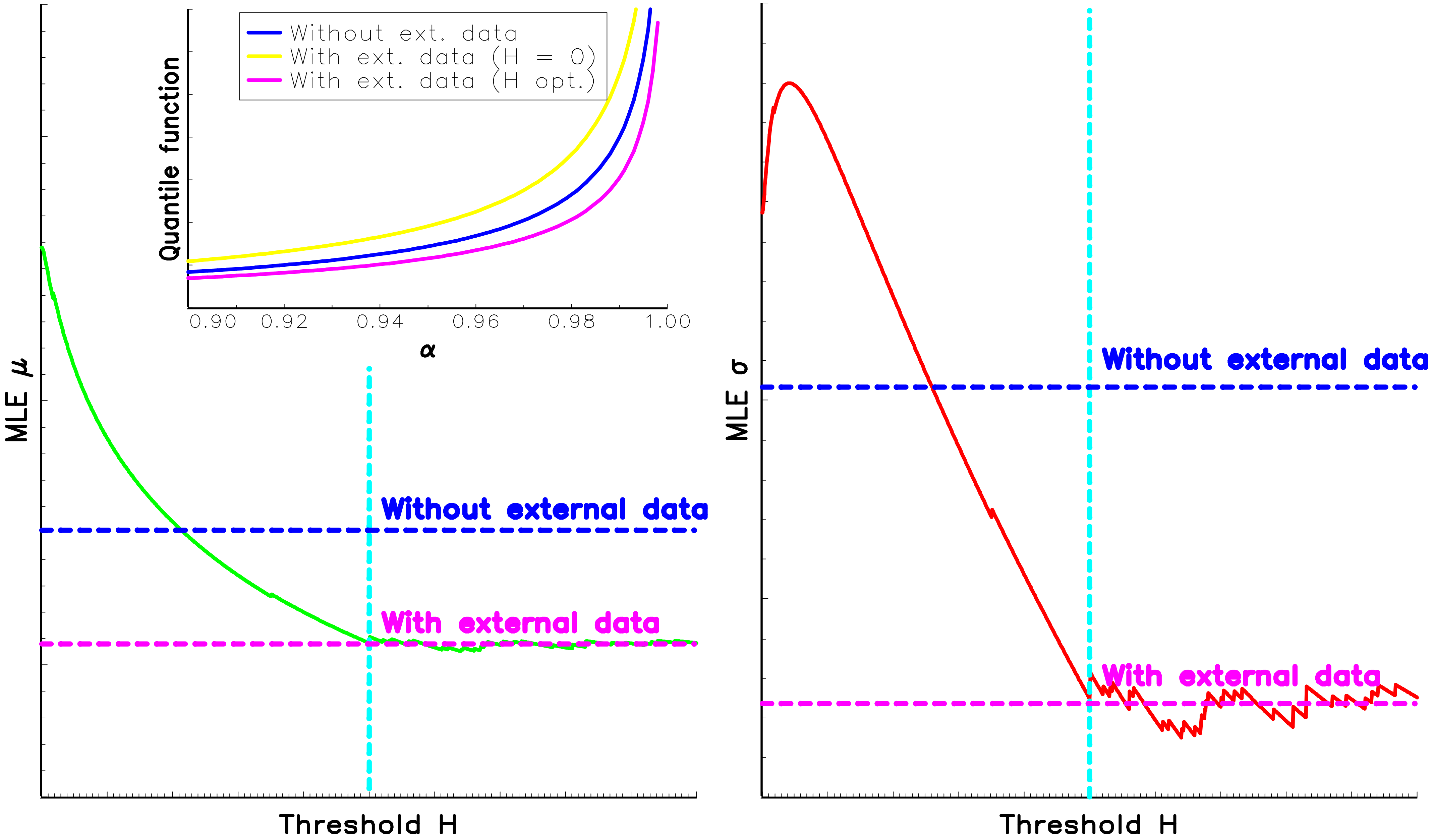
Loss type: external fraud.

The threshold H is random

Remark 6 *Banks does not use the same threshold to report losses in external databases.*

In this case, we may consider H as a random variable.

$\Rightarrow$ this approach (and the previous one with a correct formulation) is developped in a forthcoming working paper.



Mixing internal (Credit Lyonnais) and external (BBA) data
for the loss type EXTERNAL FRAUD

6 Conclusion

With respect to market and credit risk, banks have (had?) little experience about operational risk modelling.

Banks have made great progress, but there remains a lot of work to have a robust methodology for 31/12/2006.