

# CREST

## CHAIRE AXA : ASSURANCE ET RISQUES MAJEURS

Cours de Formation par la Recherche 2009-2010

### Large Portfolio, Concentration and Granularity Theory

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The risk analysis in portfolios of credits, or life insurance contracts, is made difficult by the nonlinearities of risk models, the dependencies between the individual risks, and the large size of the portfolios, which may include several hundred thousands of contracts. The granularity principle has been introduced in the Basel II regulation for credit risk to solve these difficulties when computing the reserves. The principle requires three steps. First, the modelling step considers a Risk Factor Model (RFM), which distinguishes the systematic risk from the individual specific risks. Second, this model is applied to a virtual portfolio of infinite size, leading to the so-called Asymptotic Risk Factor Model (ARFM). This gives in general explicit formulas for the Value-at-Risk (VaR), and thus for the required capital. Finally, for a portfolio of large finite size, closed form approximations are derived from an expansion around the ARFM. This provides the granularity adjustments for the required capital. In fact, the granularity principle can be applied to a variety of frameworks. It can be applied for instance for efficient estimation in panel factor models with micro- and macro-dynamics, for improving macro-predictions from micro-data, or for pricing derivatives written on large portfolios. The aim of this lecture is to provide a first overview of granularity theory by following a progressive approach

#### Outline

- Granularity in a linear dynamic framework: The linear RFM, parameter estimation, mean-variance portfolio management in the linear framework
- Local analysis of risk measures: VaR, TailVaR and Distortion Risk Measures, their expansions with respect to the risk distribution, computation of required capital in Merton's model
- Efficient estimation of micro- and macro-parameters in RFM models: Approximated likelihood, asymptotic efficiency of the fixed effect estimator and asymptotic linearization of nonlinear filters
- Stochastic migration models: rating histories, basic rating migration model, linearized version, link with the business cycle
- Derivatives written on large portfolios: CDO's, options on iTraxx and CDX, longevity bonds, closed form approximate pricing formulas

#### Bibliographie

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|-----------|-------|----|--------------|-----------------|----------------|-------|
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|           | Lundi | 25 | Janvier 2010 | De 14h à 16h 10 | C. Gouriéroux  | Salle |
|           | Lundi | 8  | Février 2010 | De 14h à 16h 10 | P. Gagliardini | Salle |
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